Modal-based Damage Localisation

BGCE research day Presentation

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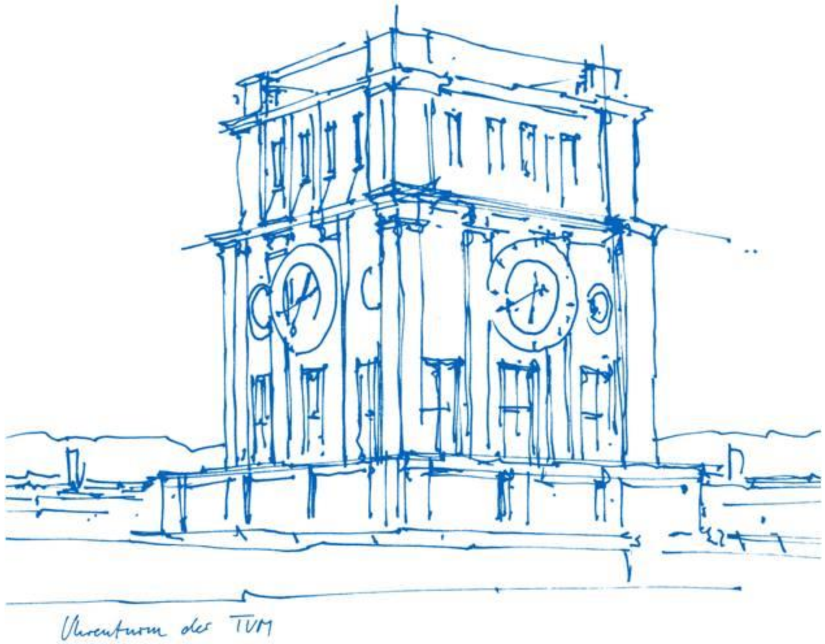


Outline

- **Part1 Introduction:** Damage Localisation + Modal based

- **Part2 Workflow:**

  benchmark tool to define a simulated damage

  → generate sensor-like data

  → try out different damage Index

  → assess damage

- **Part3 Damage assessment**
- **Part4 Summary**

Part1 Introduction: **Modal-based Damage Localisation**

Structure Health Monitoring = Damage Identification :

1.Four levels:

1).Detection (Does the damage exist?)

2).Localisation (Where is the damage?)

3).Classification & quantification (What type the damage is ? & how severe it is?)

4).Prediction (How long will the structure survive? What actions will we take?)

2.Two approaches: Choose a damage sensitive feature (DSF):

- Model-based approach (inverse-problem approach):
  - physical-based model (e.g. a FE model). First build a model, then update it using the measured datas from the real structure. Damage Index can be chosen as features from basic modal properties
- Data-based approach :
  - not based on a law-based model but a statistical model (e.g. a probability density function) from training data. Then use the pattern recognition to find the damage Index.

3. Several Axioms[1]:

- Damage can not be directly measured (Acclerations, strains, even modes-related datas can be measured)
- Damage assessment required a comparison between two system states …

1. Farrar, Charles R., and Keith Worden. *Structural health monitoring: a machine learning perspective*. John Wiley & Sons, 2012.

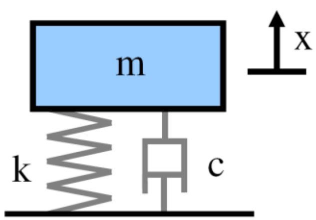
Part1 Introduction: **Modal-based Damage Localisation**

Modal analysis:

Structure (discrete or continuous system) → Dynamic analysis → **Basic Modal Properties:** Eigenvalue(resonance frequency) Eigenvector(modes or modeShapes)

↓

Derived Modal Properties:
ModeShapeCurvature
ModalStrainEnergy
FlexibilityMatrix



undamaged structure

$$[\mathrm{M}]\ddot{\boldsymbol{y}} + [C]\dot{\boldsymbol{y}} + [K]\boldsymbol{y} = \boldsymbol{f}$$

damaged structure

$$[\mathrm{M}]\ddot{\boldsymbol{y}} + [C]\dot{\boldsymbol{y}} + [K_d]\boldsymbol{y} = \boldsymbol{f}$$

Damage(**stiffness reduction):** $[K] \to [K_d]$, $[M] \to [M]$, $[C] \to [C]$

Part 2 Workflow: **Damage Settings explanation**

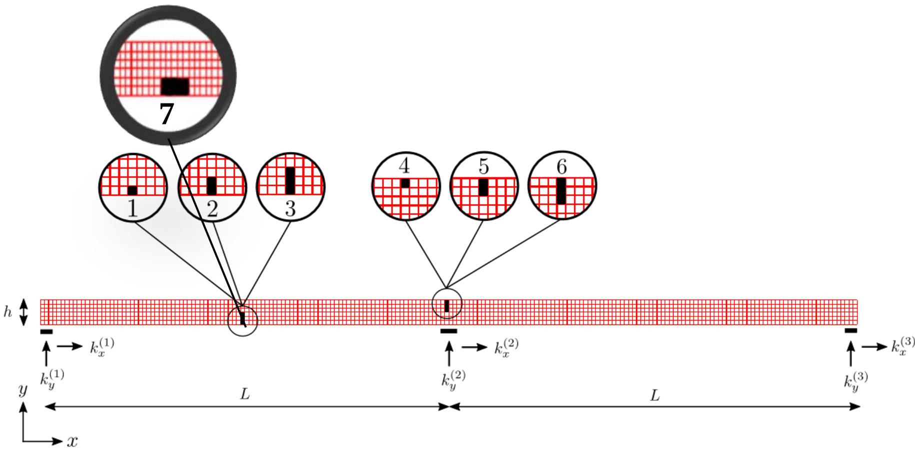


Figure 1: Finite element model of the benchmark structure.

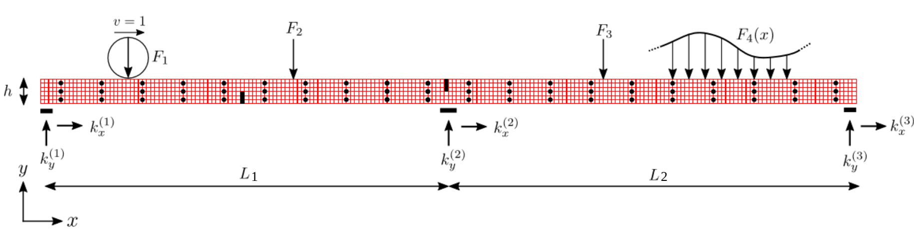


Figure 2: Load cases and measurement positions

Geometry: L1 = 12,L2=13,h=0.6,w = 0.1

FEM model: isoparametric quadrilateral elements for plane-stress problem with predefined 200x6 elements.

dx = 25/200 dy = 0.6/6

Mateial assumption :linear elastic material with E = 30e10, poisson ratio $\gamma = 0.3$, material density $\rho = 7800$

E is reduced by 20%,50%,70% at the predefined location. Here we study the case when one damage appears in the structure. Damaged7(enlarged damaged 2)

Boundary Condition: Here only F4(x) the distributed loads modeled as independent Gaussian white noise is applied.

Sensor Location : 60 sensors is predifined in the simulator, each sensor can return the value (either accelerations or modes) in x, y directions

deltax = 10*dx(20 sensors)

deltax = 20*dx(10 sensors)

Modal-based Damage Localisation| Tatsis, Konstantinos, and Eleni Chatzi. "A numerical benchmark for system identification under operational and environmental variability." *8th International Operational Modal Analysis Conference (IOMAC 2019)*. 2019.

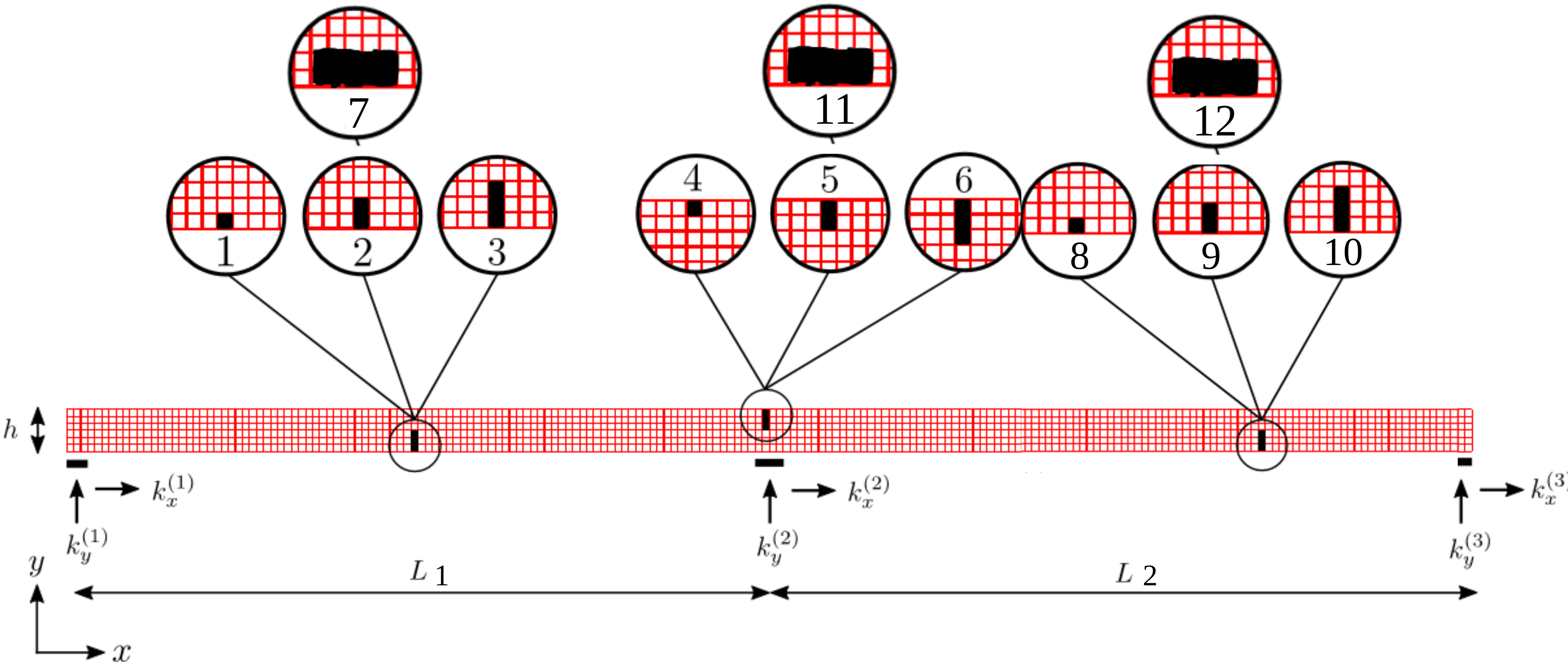
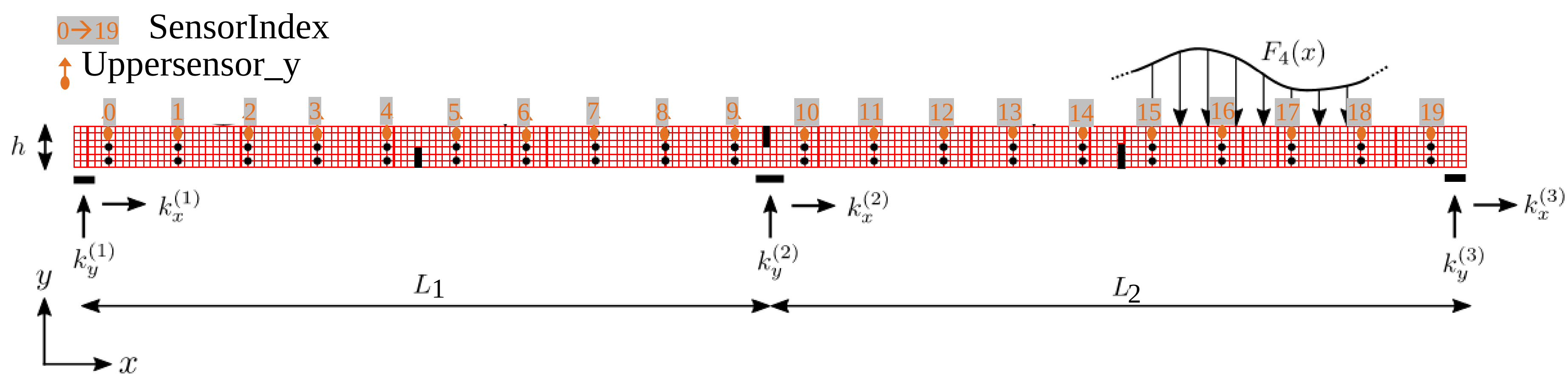
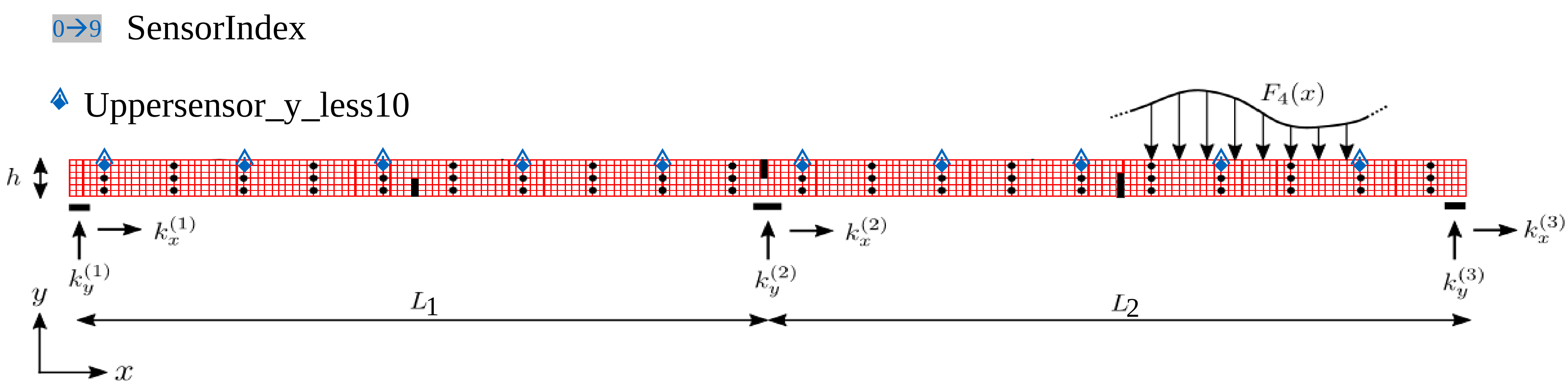
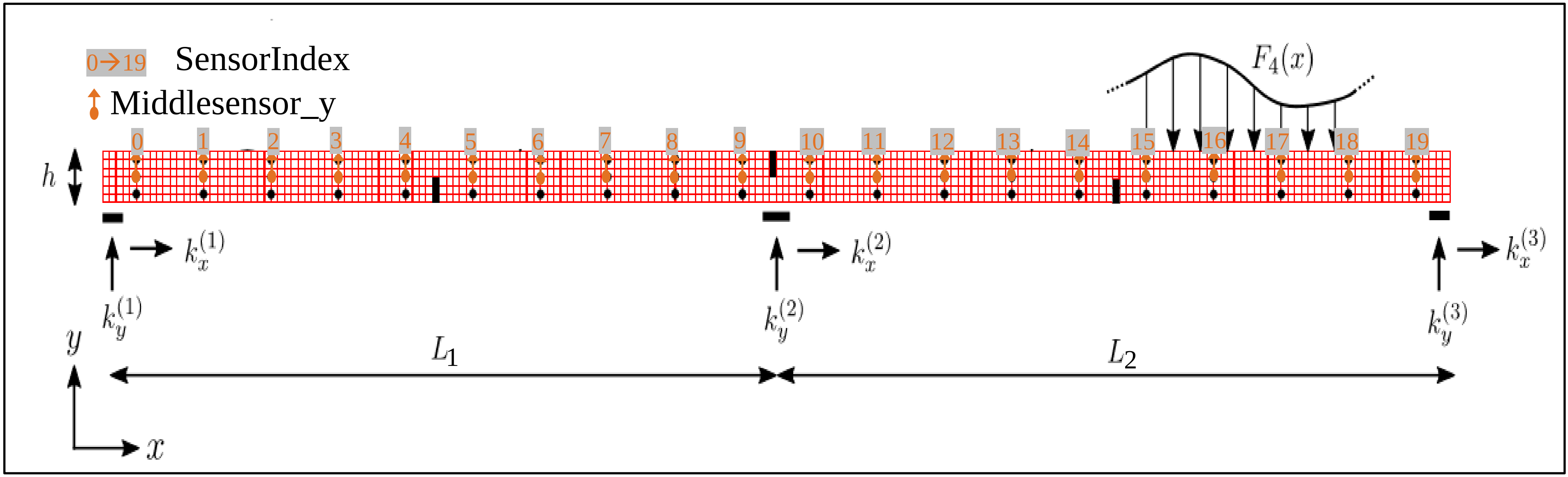


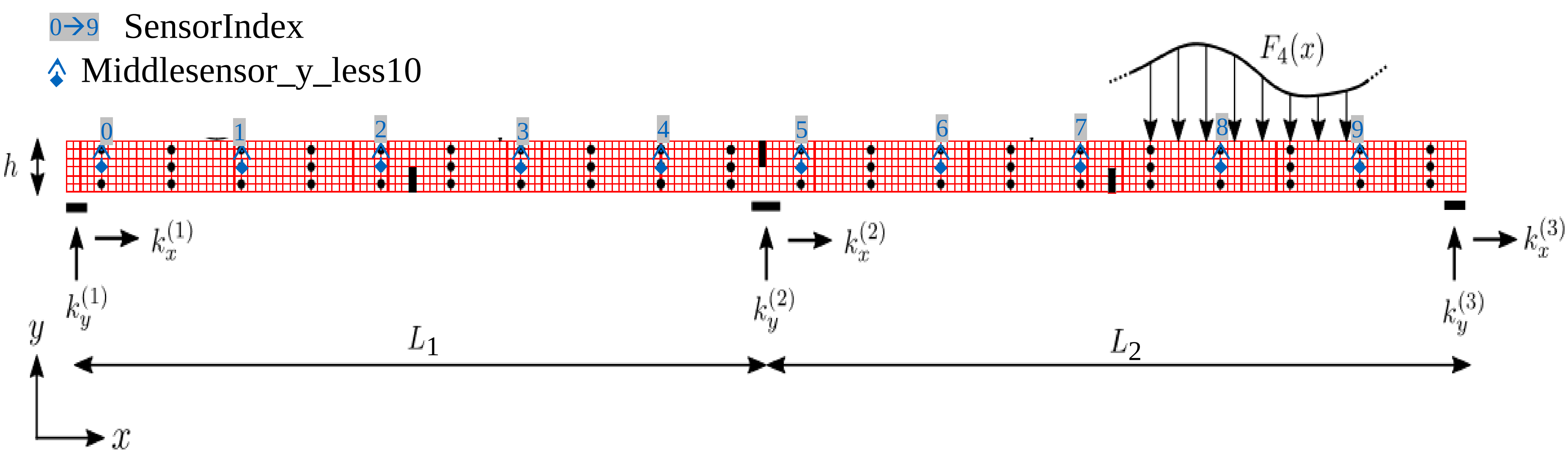
Figure 1: Finite element model of the benchmark structure.


0→19 SensorIndex
Uppersensor_y
$F_4(x)$
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
h
$k_x^{(1)}$
$k_y^{(1)}$
$k_x^{(2)}$
$k_y^{(2)}$
$k_x^{(3)}$
$k_y^{(3)}$
L_1
L_2
y
x

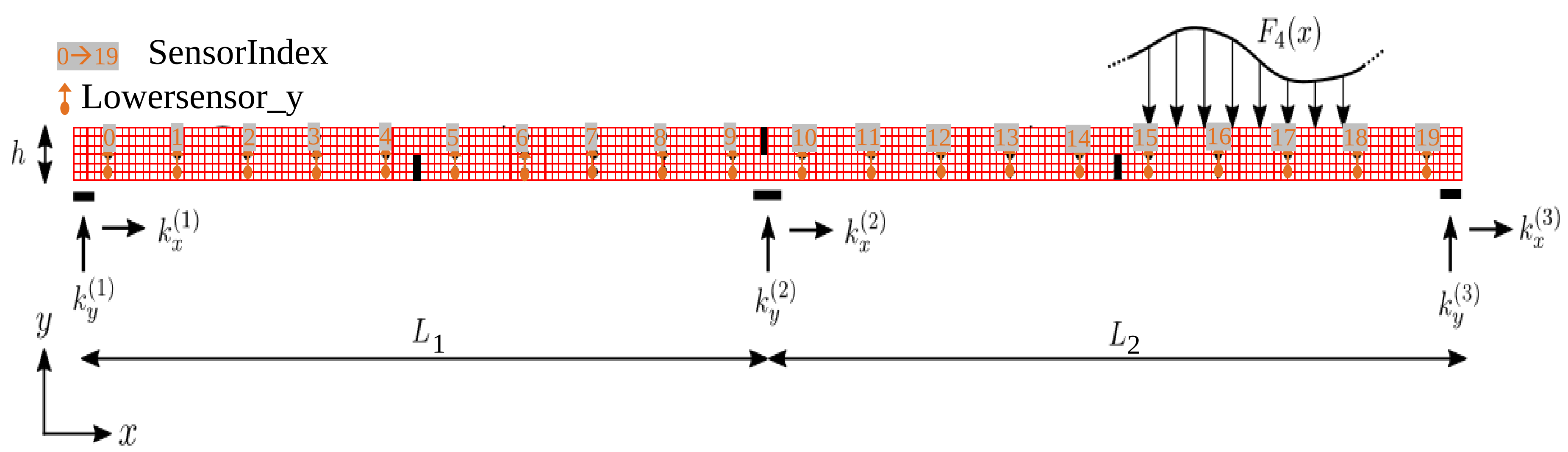

0→9 SensorIndex
Uppersensor_y_less10
$F_4(x)$
h
$k_x^{(1)}$
$k_y^{(1)}$
$k_x^{(2)}$
$k_y^{(2)}$
$k_x^{(3)}$
$k_y^{(3)}$
L_1
L_2
y
x



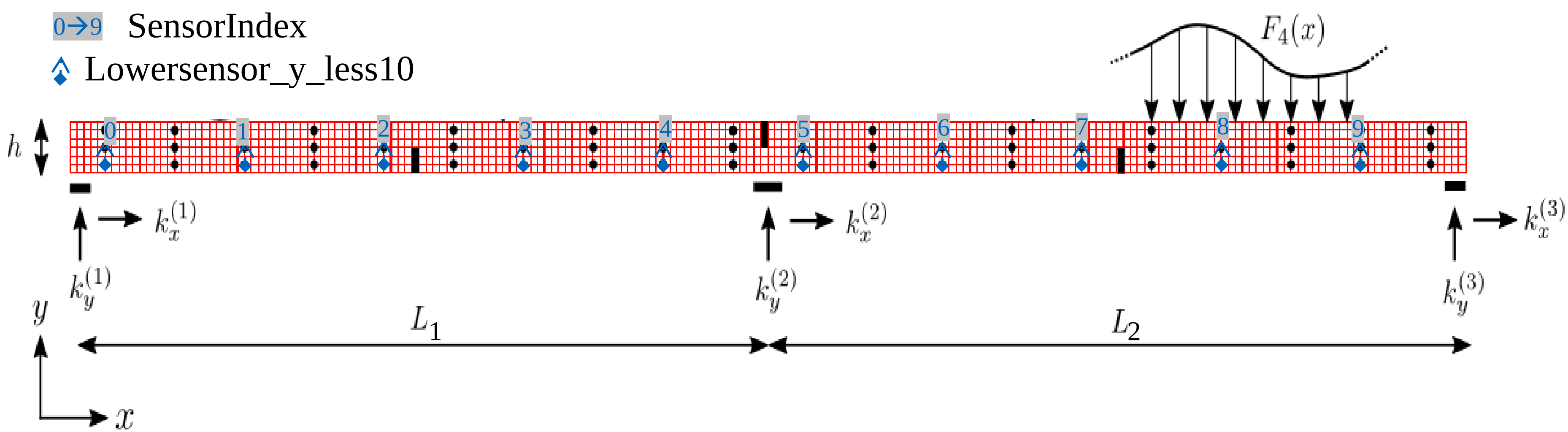
0→19 SensorIndex
Middlesensor_y
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
$F_4(x)$
h
$k_x^{(1)}$ $k_x^{(2)}$ $k_x^{(3)}$
$k_y^{(1)}$ $k_y^{(2)}$ $k_y^{(3)}$
L_1 L_2
y x



0→9 SensorIndex
Middlesensor_y_less10
0 1 2 3 4 5 6 7 8 9
$F_4(x)$
h
$k_x^{(1)}$ $k_x^{(2)}$ $k_x^{(3)}$
$k_y^{(1)}$ $k_y^{(2)}$ $k_y^{(3)}$
L_1 L_2
y x



0→19 SensorIndex
Lowersensor_y
$F_4(x)$
h
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
$k_x^{(1)}$ $k_x^{(2)}$ $k_x^{(3)}$
$k_y^{(1)}$ $k_y^{(2)}$ $k_y^{(3)}$
L_1 L_2
y x



0→9 SensorIndex
Lowersensor_y_less10
$F_4(x)$
h
0 1 2 3 4 5 6 7 8 9
$k_x^{(1)}$ $k_x^{(2)}$ $k_x^{(3)}$
$k_y^{(1)}$ $k_y^{(2)}$ $k_y^{(3)}$
L_1 L_2
y x

Combine the sensor and damage settings

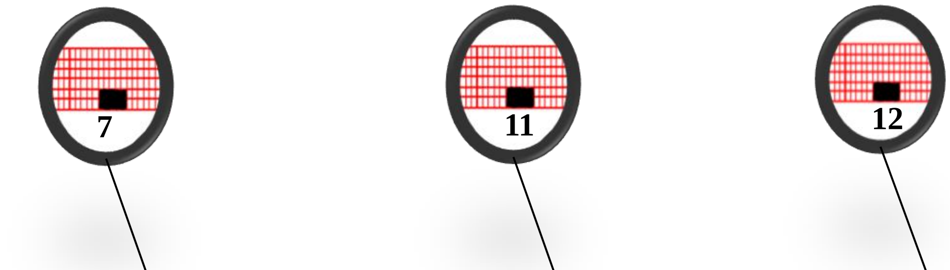


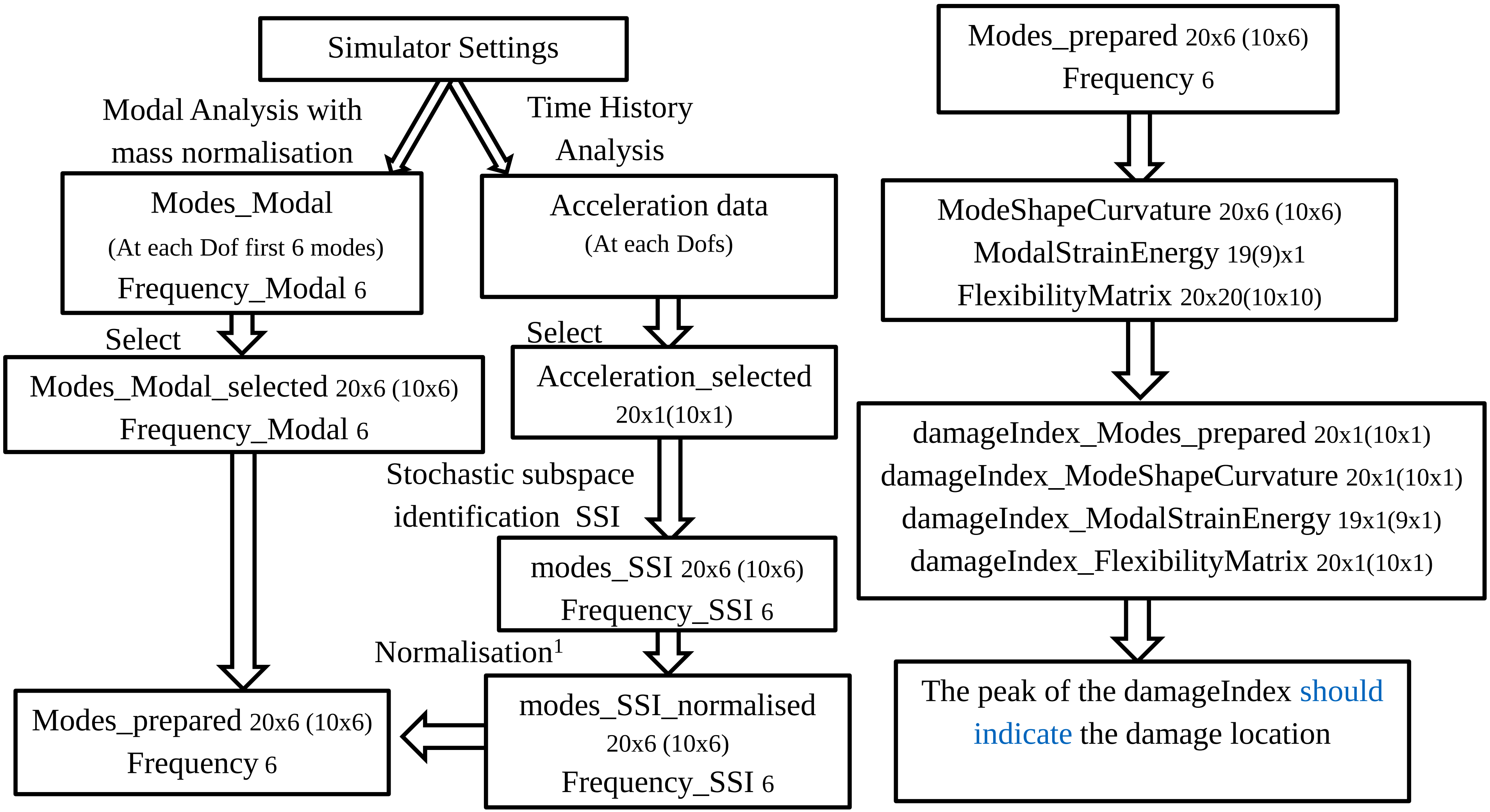
Table1 20 Sensors index

Region Index		0	1	2	3	**4**	5	6	7	8	**9**	10	11	12	13	**14**	15	16	17	18	
Sensor Index	Uppersensor_y Middlesensor_y Lowersensor_y	0	1	2	3	**4**	**5**	6	7	8	**9**	**10**	11	12	13	**14**	**15**	16	17	18	19
x (*dx)	dx = 25/200	5	15	25	35	45	55	65	75	85	**95**	**105**	115	125	135	**145**	**155**	165	175	185	195

Table2 10 Sensors index

Region Index		0	1	**2**	3	4	5	6	7	8	
Sensor Index	Uppersensor_y_less10 Middlesensor_y_less10 Lowersensor_y_less10	0	1	2	3	**4**	**5**	6	**7**	**8**	9
x (*dx)	dx = 25/200	5	25	45	65	**85**	**105**	125	**145**	**165**	185

Part 2 Workflow **summary**



Part 3 Damage assessment: **damageIndex overview**

To define a damage index, we need to find the damage sensitive features (DSF)
And we know that damage assessment is between two states, we need to make an appropriate **mathematical operations** to see the difference between healthy and unhealthy state:

#Subtraction $\mathrm{DamageIndex} = \mathrm{DSF}_{\mathrm{damaged}} - \mathrm{DSF}_{\mathrm{undamaged}}$

1. $\mathrm{DamageIndex\_absdiff} = \mathrm{abs}\left(\mathrm{DSF}_{\mathrm{damaged}} - \mathrm{DSF}_{\mathrm{undamaged}}\right)$
2. $\mathrm{DamageIndex\_absabsdiff} = \mathrm{abs}\left(\mathrm{abs}\left(\mathrm{DSF}_{\mathrm{damaged}}\right) - \mathrm{abs}\left(\mathrm{DSF}_{\mathrm{undamaged}}\right)\right)$
3. $\mathrm{DamageIndex\_abssqudiff} = \mathrm{abs}((\mathrm{DSF}_{\mathrm{damaged}})^2 - (\mathrm{DSF}_{\mathrm{undamaged}})^2)$

#Division

$$\mathrm{DamageIndex\_division} = \frac{\mathrm{DSF}_{\mathrm{damaged}}}{\mathrm{DSF}_{\mathrm{undamaged}}}$$

Part 3 Damage assessment :**DamageIndex basic modal Features**

Frequency :

Frequency_Modal	undamaged	damaged7_20%	damaged7_50%	damaged7_70%
1st	7.6763	7.6698	7.6532	7.6309
2nd	9.3719	9.3555	9.3156	9.2661
3rd	19.5276	19.5261	19.5224	19.5177
4th	23.4582	23.4565	23.4524	23.4470
5th	34.1261	34.0891	33.9953	33.8713
6th	38.2315	38.1934	38.1017	37.9909

Mode shape:

Frequency_SSI	undamaged	damaged7_20%	damaged7_50%	damaged7_70%
1st	7.6797	7.6686	7.6488	7.6234
2nd	9.3689	9.3500	9.3150	9.2689
3rd	19.5221	19.5127	19.5146	19.5095
4th	23.4183	23.4466	23.4106	23.4064
5th	34.0940	33.9948	33.9075	33.7713
6th	38.1451	38.1160	37.9812	37.8777

Frequency_SSI_less10	undamaged	damaged7_20%	damaged7_50%	damaged7_70%
1st	7.6797	7.6686	7.6488	7.6234
2nd	9.3689	9.3500	9.3150	9.2689
3rd	19.5221	19.5127	19.5146	19.5095
4th	23.4183	23.4466	23.4105	23.4064
5th	34.0947	33.9948	33.9078	33.7713
6th	38.1466	38.1153	37.9802	37.8787

Part 3 Damage assessment :**DamageIndex basic modal Features**

Frequency:

Mode shape:

Modes_Modal(modes_SSI):

~~absdiff☹☹☹~~ absdiff $=abs(\mathrm{DSF}_{\mathrm{damaged}} - \mathrm{DSF}_{\mathrm{undamaged}})$

#each order

#Uppersensor_y(20sensors) #Uppersensor_y_less10(10sensors)

#Sum of all order

#Uppersensor_y(20sensors) #Uppersensor_y_less10(10sensors)

#Sum of first two orders

#Uppersensor_y(20sensors) #Uppersensor_y_less10(10sensors)

absabsdiff☹ absabsdiff $=abs(abs(\mathrm{DSF}_{\mathrm{damaged}}) - abs(\mathrm{DSF}_{\mathrm{undamaged}}))$

#each order

#Uppersensor_y(20sensors) #Uppersensor_y_less10(10sensors)

#Sum of all order

#Uppersensor_y(20sensors) #Uppersensor_y_less10(10sensors)

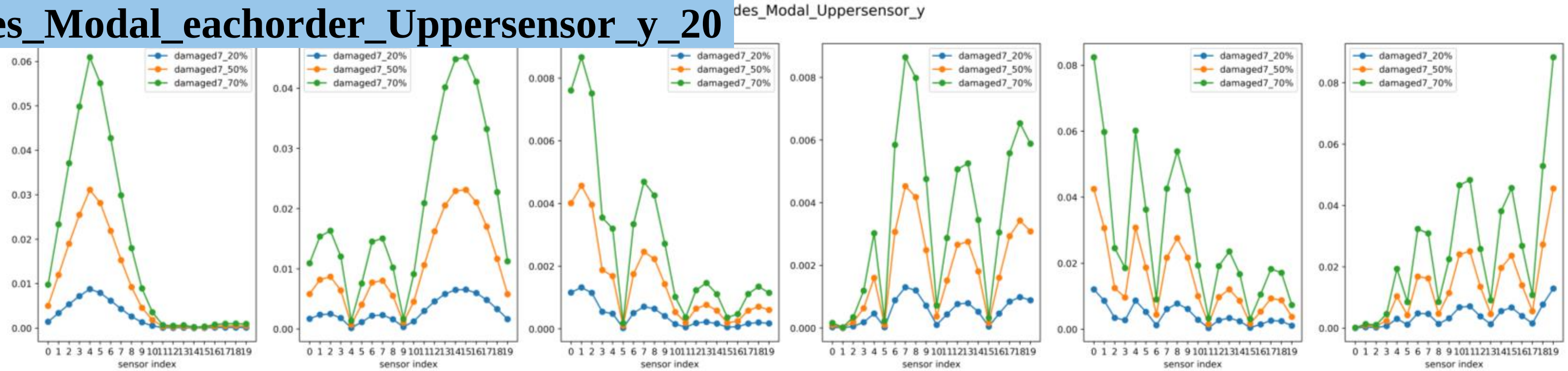
#Sum of first two orders

#Uppersensor_y(20sensors) #Uppersensor_y_less10(10sensors)

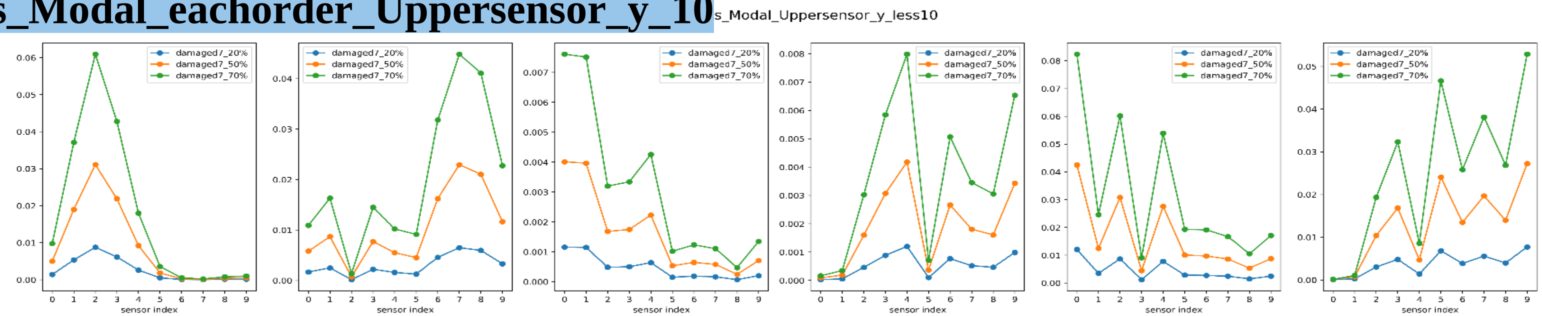
abssqudiff☹ abssqudiff $=abs((\mathrm{DSF}_{\mathrm{damaged}})^2 - (\mathrm{DSF}_{\mathrm{undamaged}})^2)$

…..

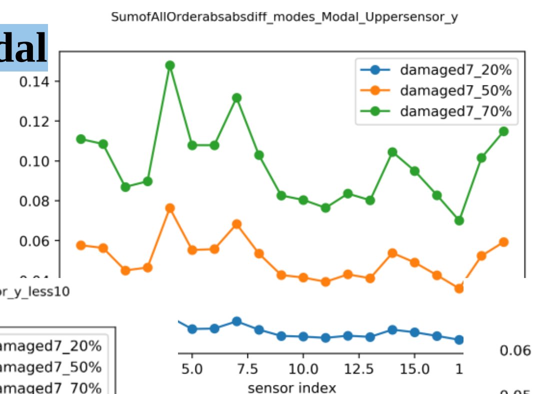
Absabsdiff_modes_Modal_eachorder_Uppersensor_y_20



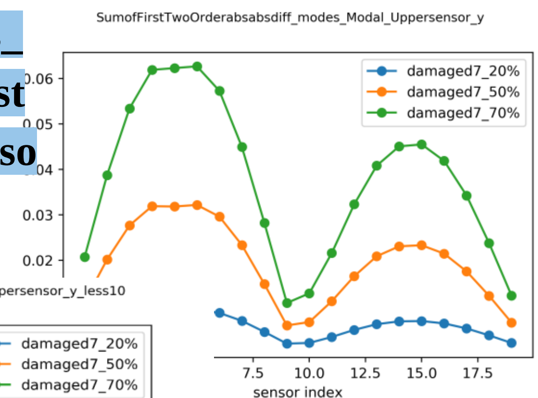
Absabsdiff_modes_Modal_eachorder_Uppersensor_y_10

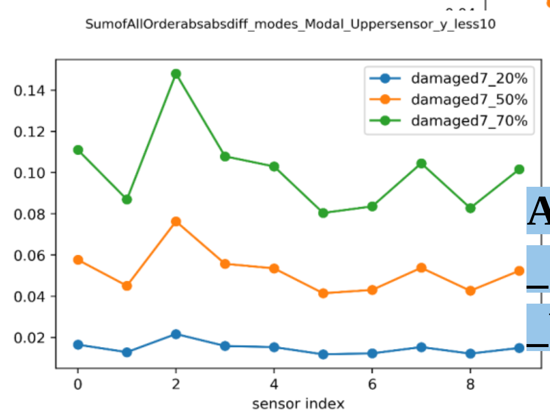


Absabsdiff_modes_Modal_SumAllOrder_Uppersensor_y_20

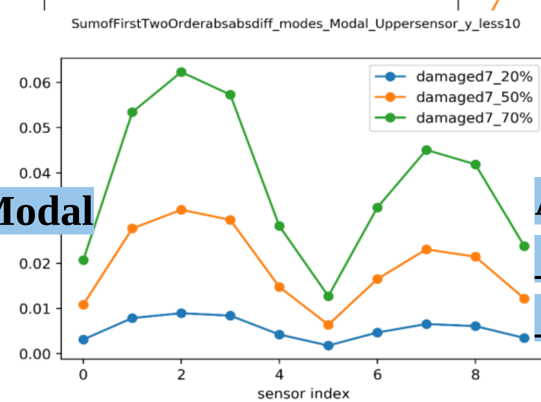


Absabsdiff_modes_Modal_SumOfFirst2Order_Uppersensor_y_20





Absabsdiff_modes_Modal_SumAllOrder_Uppersensor_y_10



Absabsdiff_modes_Modal_SumOfFirst2Order_Uppersensor_y_10

Part 3 Damage assessment :**DamageIndex derived modal Features**

ModeShapeCurvature: (2^{nd} order derivative of modeShape)

The middle points use the central difference scheme to get the 2^{nd} ModeShapeCurvature

Middle points:

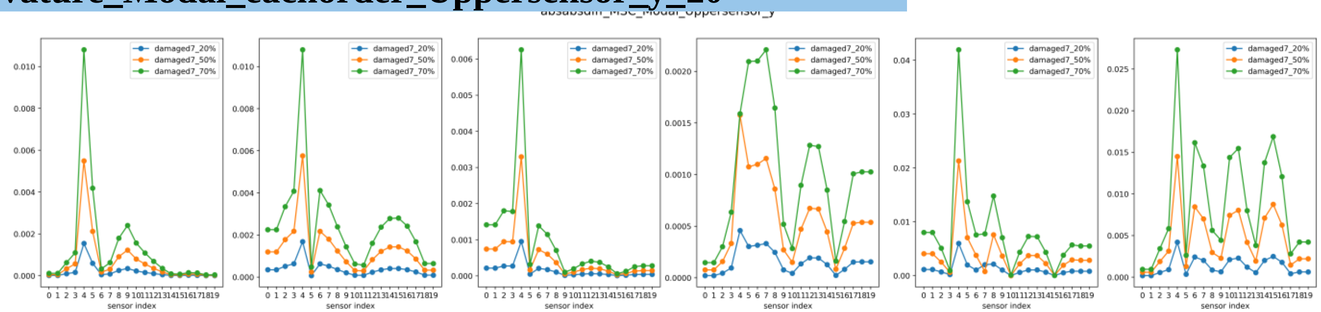
$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{i} = \frac{u_{i-1} - 2u_i + u_{i+1}}{\mathrm{deltax}^2} + o(\mathrm{deltax})$$

Boundary :

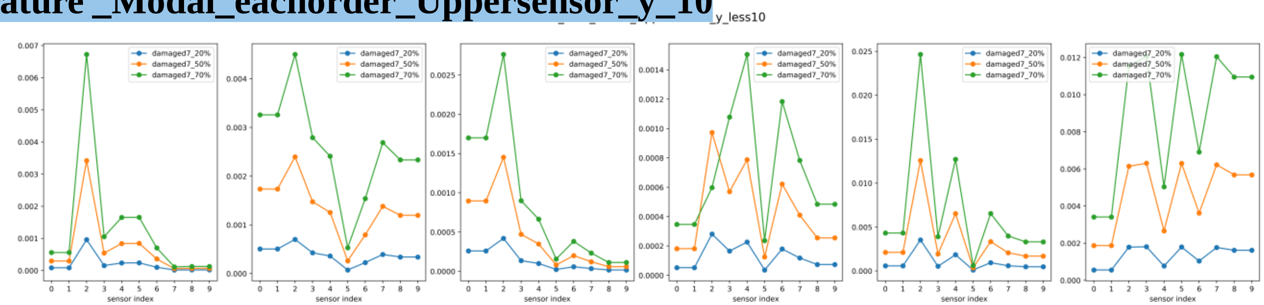
$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{i=0} = \frac{u_0 - 2u_1 + u_2}{\mathrm{deltax}^2} + o(\mathrm{deltax})$$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_{i=\mathrm{sensor}-1} = \frac{u_{\mathrm{sensor}-3} - 2u_{\mathrm{sensor}-2} + u_{\mathrm{sensor}-1}}{\mathrm{deltax}^2} + o(\mathrm{deltax})$$

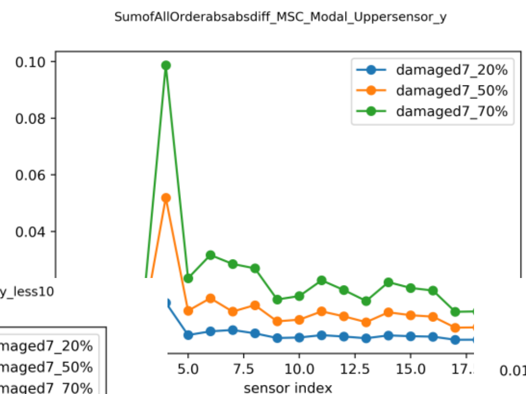
Absabsdiff_ModeShapeCurvature_Modal_eachorder_Uppersensor_y_20



Absabsdiff_ ModeShapeCurvature _Modal_eachorder_Uppersensor_y_10



Absabsdiff_ ModeShapeCurvature _Modal_SumAllOrder _Uppersensor_y_20



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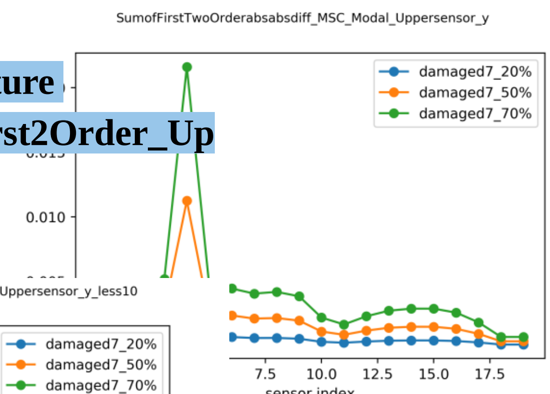
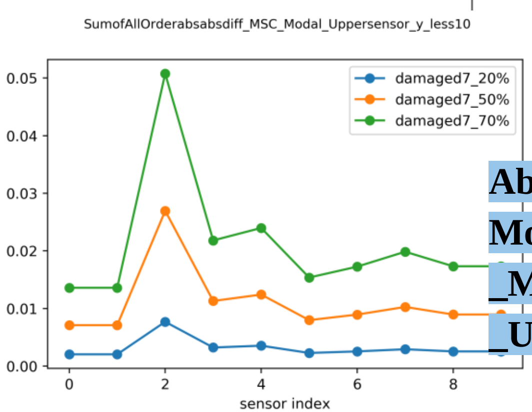
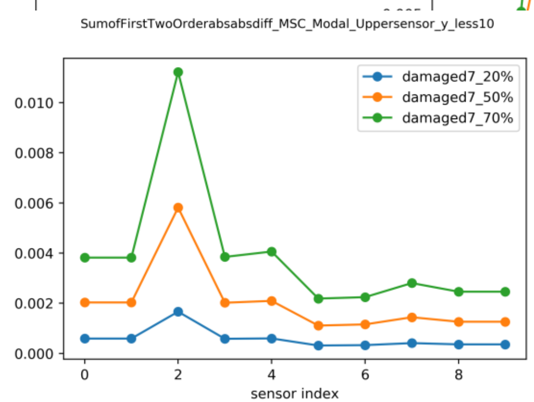


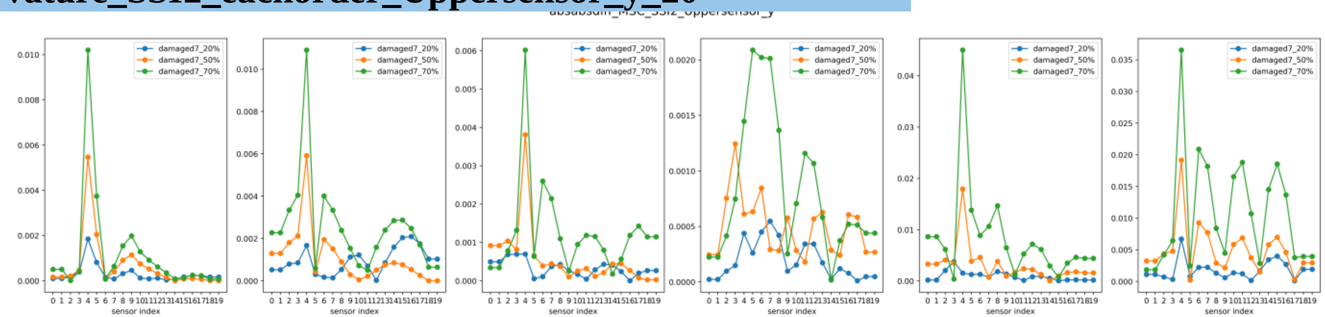


Absabsdiff_ ModeShapeCurvature _Modal_SumAllOrder _Uppersensor_y_10

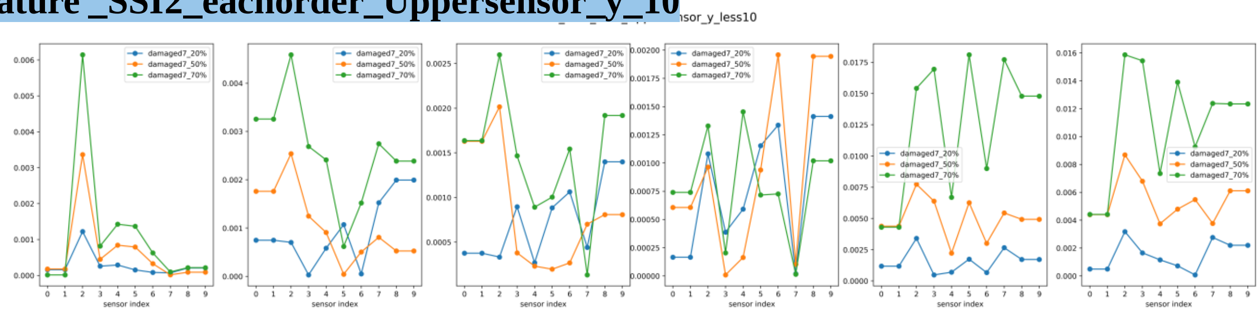


Absabsdiff_ ModeShapeCurvature _Modal_SumOfFirst2Orde _Uppersensor_y_10

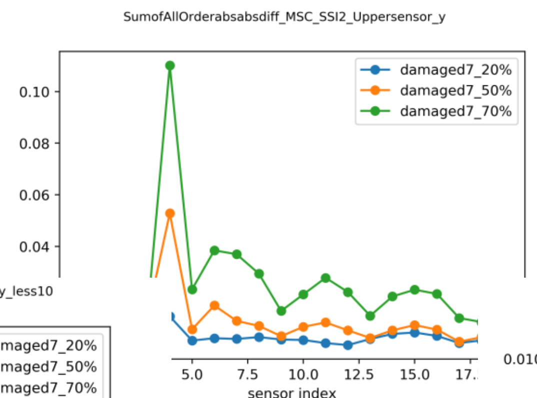
Absabsdiff_ModeShapeCurvature_SSI2_eachorder_Uppersensor_y_20



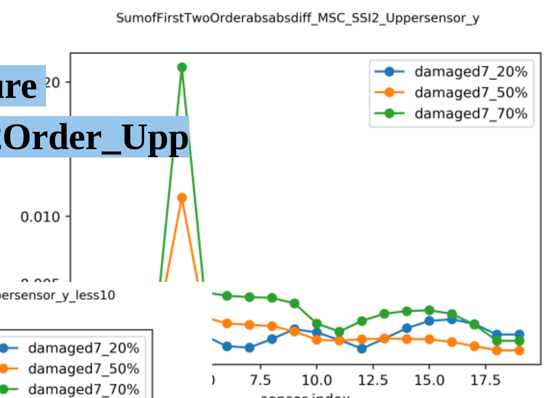
Absabsdiff_ ModeShapeCurvature _SSI2_eachorder_Uppersensor_y_10

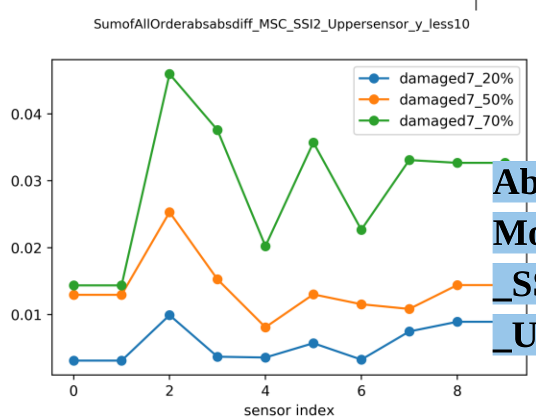


Absabsdiff_ ModeShapeCurvature _SSI2_SumAllOrder _Uppersensor_y_20

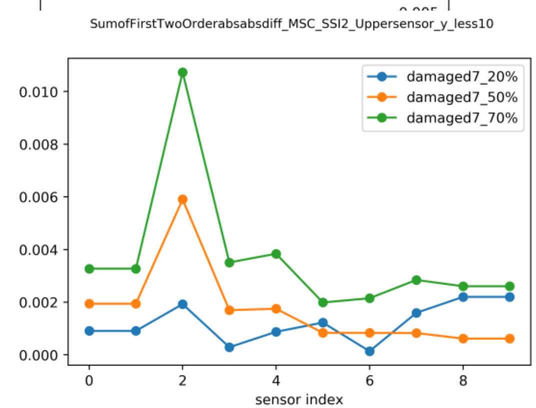


Absabsdiff_ ModeShapeCurvature _SSI2_SumOfFirst2Order_Uppersensor_y_20



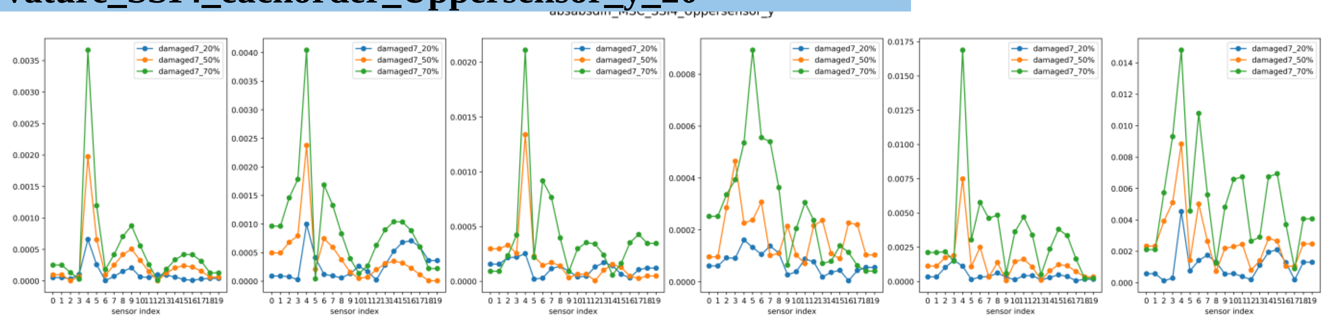


Absabsdiff_ ModeShapeCurvature _SSI2_SumAllOrder _Uppersensor_y_10

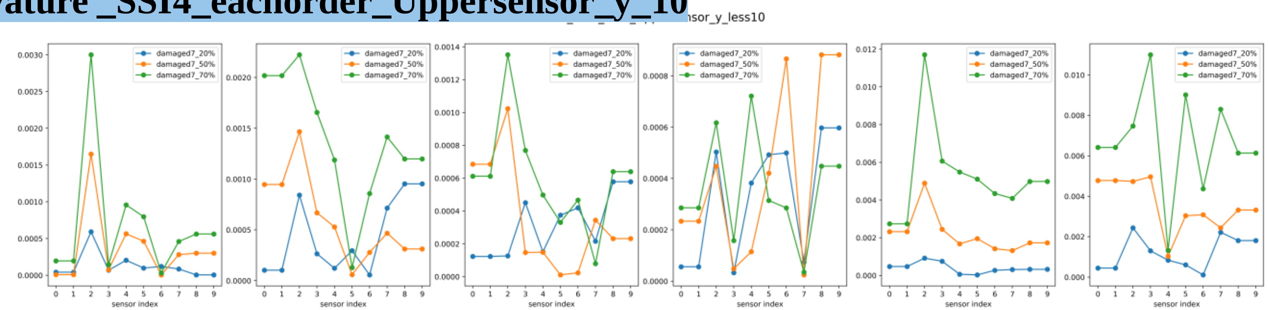


Absabsdiff_ ModeShapeCurvature _SSI2_SumOfFirst2Order _Uppersensor_y_10

Absabsdiff_ModeShapeCurvature_SSI4_eachorder_Uppersensor_y_20



Absabsdiff_ ModeShapeCurvature _SSI4_eachorder_Uppersensor_y_10



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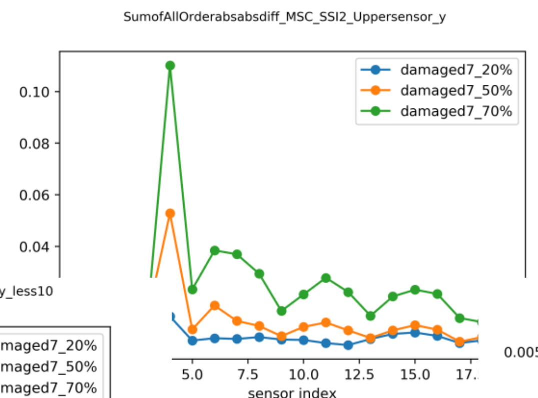

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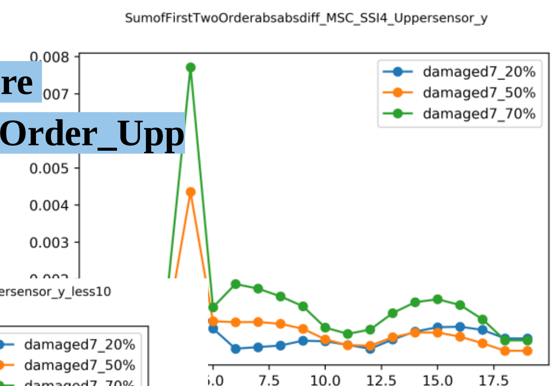
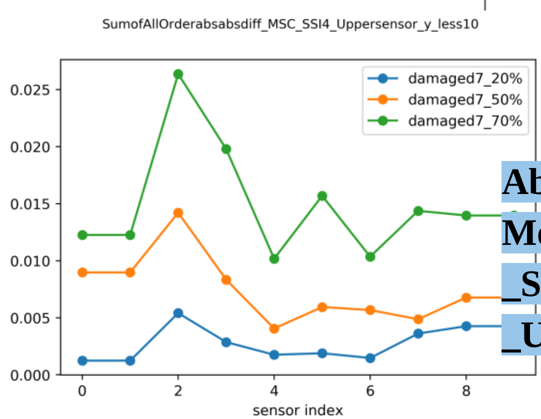
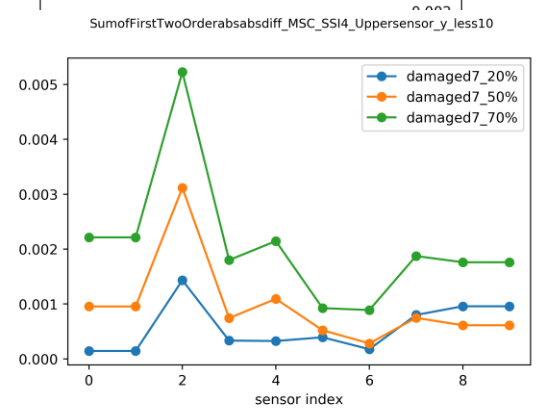




Absabsdiff_ ModeShapeCurvature _SSI4_SumAllOrder _Uppersensor_y_10



Absabsdiff_ ModeShapeCurvature _SSI4_SumOfFirst2Order _Uppersensor_y_10

Part 3 Damage Index derived modal Features

Modal Strain Energy:

The strain energy stored in the spring when the structures deform in one of its mode shapes is:

$$U = \frac{1}{2}k(\Delta x)^2, \Delta\text{x is the change of length of the springs from its undeformed state}$$

The strain energy U of a Euler-Bernoulli beam of length L is given by $U = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx,$

$w\ is\ the\ transverse\ displacement\ of\ the\ beam, x\ isthe\ coordinate\ along\ the\ length\ of\ the\ beam$

For a particular mode shape ψ_i, the strain energy associated with the deformation in that mode shape pattern is

$$U_i = \frac{1}{2}\int_0^l EI\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 dx = \frac{1}{2}EI\int_0^l \left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 dx$$

Since the beam is subdivided into N divisions as shown in the figure, then the energy associated with each subregion j due to the ith mode is given by

$$U_{ij} = \frac{1}{2}\int_{a_j}^{a_{j+1}} (EI)_j\left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 dx = \frac{1}{2}(EI)_j\int_{a_j}^{a_{j+1}} \left(\frac{\partial^2 \psi_i}{\partial x^2}\right)^2 dx$$

The fractional energy is defined as $F_{ij} = \frac{U_{ij}}{U_i}$

Part 3 Damage Index derived modal Features

Modal Strain Energy:

Schematic illustration of a beam’ s division

0	1	2	…	j	…	N

Node index	0	1	2	3 …	j	j+1 …	N	N+1	
x position	5dx	15dx	25dx	30dx…	a_j	a_{j+1}	l-15dx	l-5dx	(δ_x=10dx)
x position	5dx	25dx	45dx	65dx…	a_j	a_{j+1}	l-25dx	l-5dx	(δ_x=20dx)

Similar quantities can be defined for a damaged structure $F_{ijd} = \frac{U_{ijd}}{U_{id}}$,$\sum_{j=0}^{j=N} F_{ij} = \sum_{j=0}^{j=N} F_{ijd} = 1$

If it is assumed that the damage is primarily located at a single subregion then the fractional energy will remain relatively constant in the undamaged subregions and in the damaged region k $F_{ikd} = F_{ik}$

$$\frac{U_{ikd}}{U_{id}} = \frac{U_{ik}}{U_i} \frac{(EI)_k}{((EI)_d)_k} = \frac{\dfrac{\int_{a_k}^{a_{k+1}} \left(\frac{\partial^2 \psi_{i,d}}{\partial x^2}\right)^2 dx}{\int_0^l \left(\frac{\partial^2 \psi_{id}}{\partial x^2}\right)^2 dx}}{\dfrac{\int_{a_k}^{a_{k+1}} \left(\frac{\partial^2 \psi_{i}}{\partial x^2}\right)^2 dx}{\int_0^l \left(\frac{\partial^2 \psi_{i}}{\partial x^2}\right)^2 dx}} \equiv \frac{f_{ikd}}{f_{ik}} \quad \text{damage Index } \beta_k = \frac{\sum_{i=1}^{i=m} f_{ikd}}{\sum_{i=1}^{i=m} f_{ik}}$$

Part 3 Damage Index derived modal Features

Modal Strain Energy:

$$\text{damage Index sum of all order: } \beta_k = \frac{\sum_{i=1}^{i=m} f_{ikd}}{\sum_{i=1}^{i=m} f_{ik}}$$

$$\text{sum of first two order } \beta_{2k} = \frac{\sum_{i=1}^{i=2} f_{ikd}}{\sum_{i=1}^{i=2} f_{ik}}$$

Fractional energy f_modal(f_SSI):

beta:

Uppersensor_y_20 # Uppersensor_y_10

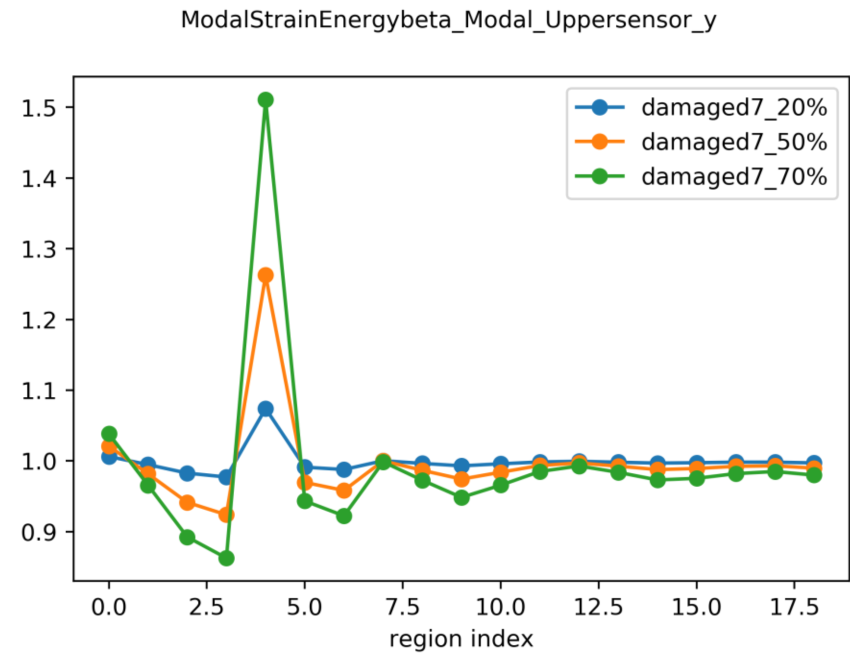
beta2:

#Uppersensor_y_20 #Uppersensor_y_10

The cubic spline interpolation is done for the modes in python:

The deviation is analytically

The integration is obtained by the summation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damaged7_50%
damaged7_70%
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1.4
1.3
1.2
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0.9
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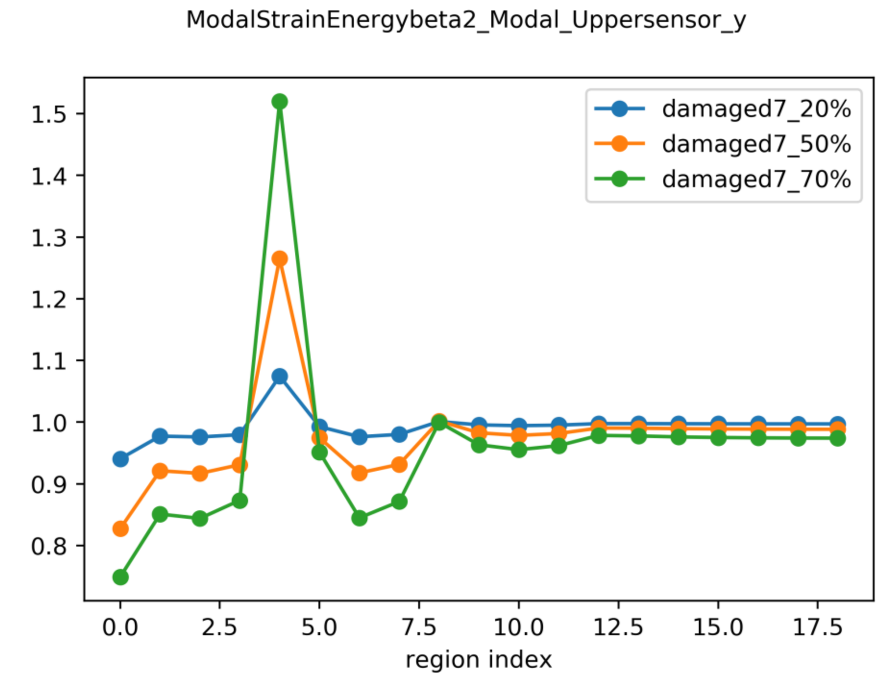
7.5
10.0
12.5
15.0
17.5
region index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damaged7_70%
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1.3
1.2
1.1
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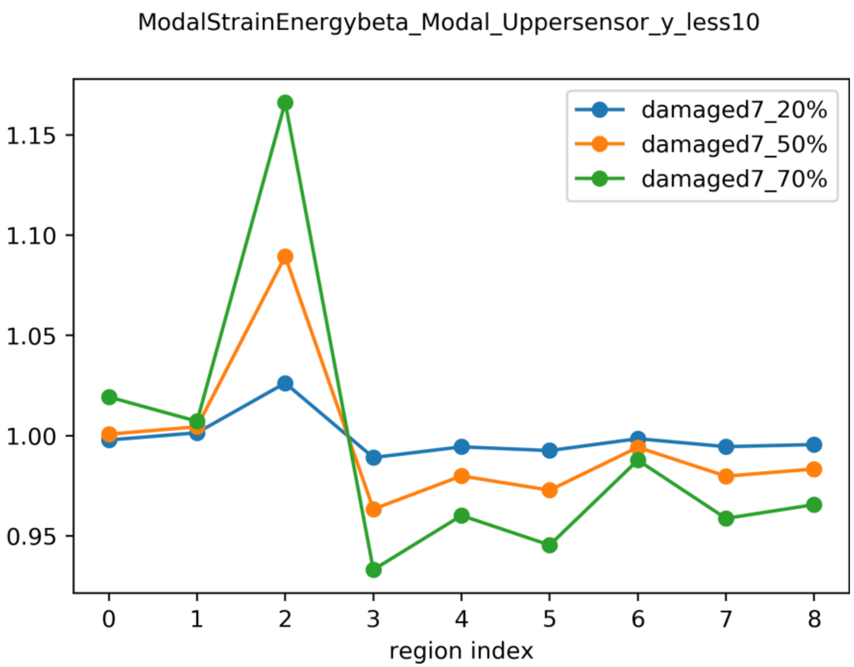
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
region index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damaged7_50%
damaged7_70%
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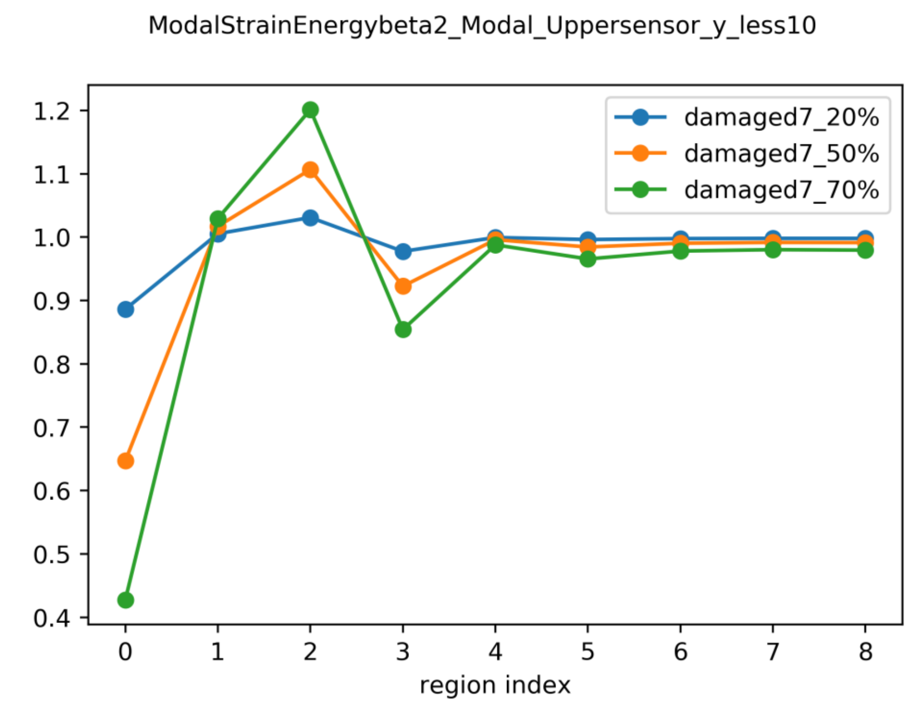
1
2
3
4
5
6
7
8
region index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0.4
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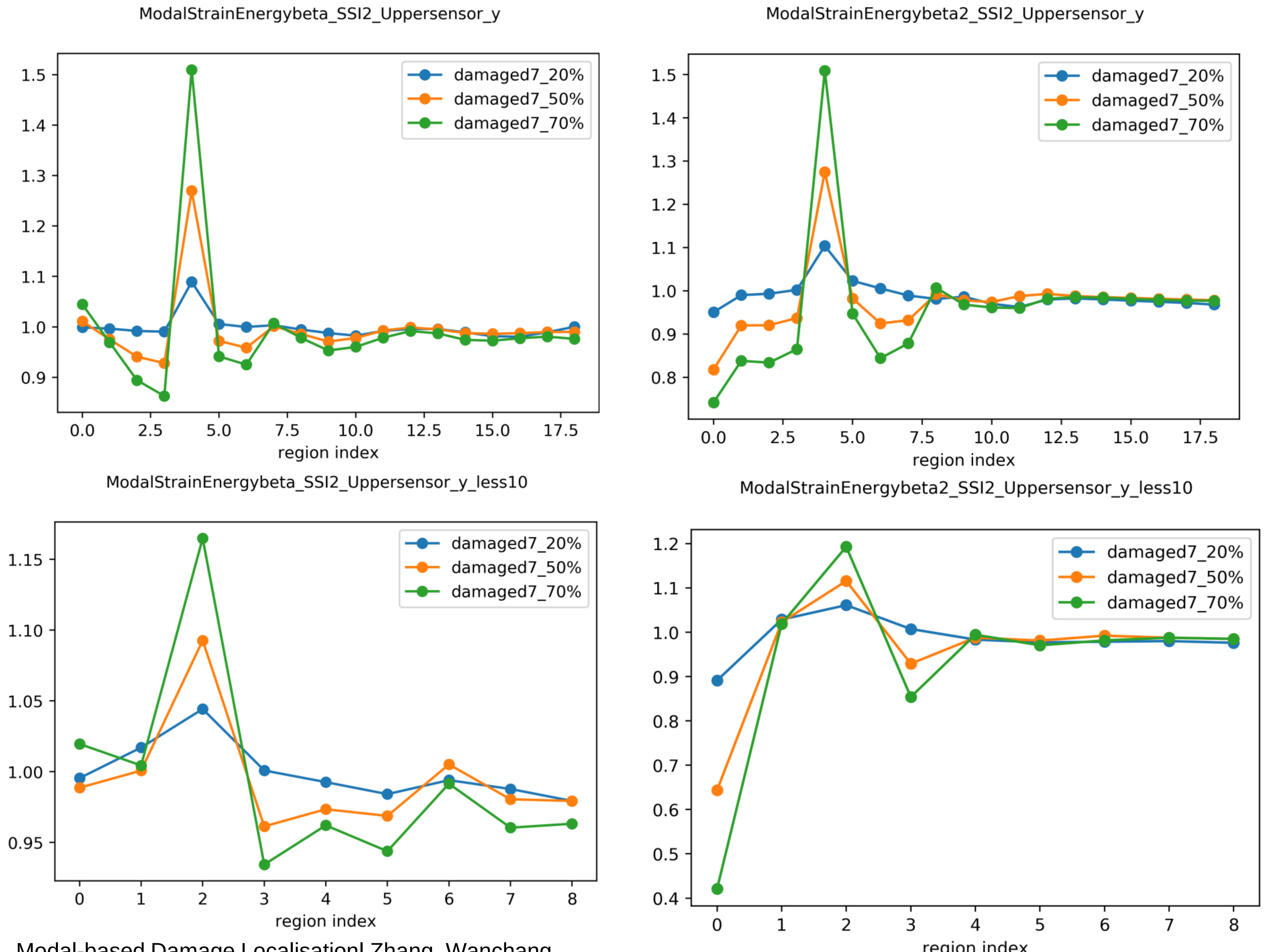
6
7
8
region index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ModalStrainEnergybeta2_SSI2_Uppersensor_y
ModalStrainEnergybeta_SSI2_Uppersensor_y_less10
ModalStrainEnergybeta2_SSI2_Uppersensor_y_less10
damaged7_20%
damaged7_50%
damaged7_70%
region index

Part 3 Damage Index derived modal Features

Flexibility Matrix:

$$[f] = [\mathrm{k}][\mathrm{y}]$$

$$[y] = [k]^{-1}[\mathrm{f}] = [\mathrm{G}][\mathrm{f}]$$

[f] is the vector of static loads applied to the structure and

[y] is a vector corresponding to the deformation

For an undamged structure with m **mass-normalised** modal vectors identified from experimental data obtained at n degrees of freedom, the $n \times n$ flexibility matrix [G] can be derived from the modal data as follows[1]:

$$[\mathrm{G}] \approx [\Phi][\Lambda][\Phi]^T \approx \sum_{i=1}^{i=m} \frac{1}{{\omega_i}^2}[\psi]_i\,{[\psi]_i}^T$$

$$[\Phi] = \text{the mode shape matrix} = \left[[\psi]_1, [\psi]_2, \dots, [\psi]_m\right],$$

$$[\psi]_i\ = \text{the i}^{\text{th}}\text{ mass normalised mode shape},$$

$$[\Lambda] = \text{the modal stiffness matrix} = \mathrm{diag}\left({\omega_i}^2\right),$$

$$\omega_i = the\ i^{th}\ \ modal\ frequency$$

Part 3 Damage Index derived modal Features

Flexibility Matrix:

$$[\mathrm{G}] \approx [\Phi][\Lambda][\Phi]^T \approx \sum_{i=1}^{i=m} \frac{1}{{\omega_i}^2} [\psi]_i \, [\psi]_i{}^T$$

For each order define:

$$[\mathrm{g}] \approx \frac{1}{{\omega_i}^2} [\psi]_i [\psi]_i{}^T$$

1st mode 2nd mode … mth mode

$[\mathrm{g}_{\text{undamaged}}]$
$[\mathrm{g}_{\text{damaged}}]$

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}_{n\times n} \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}_{n\times n} \dots \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}_{n\times n}$$

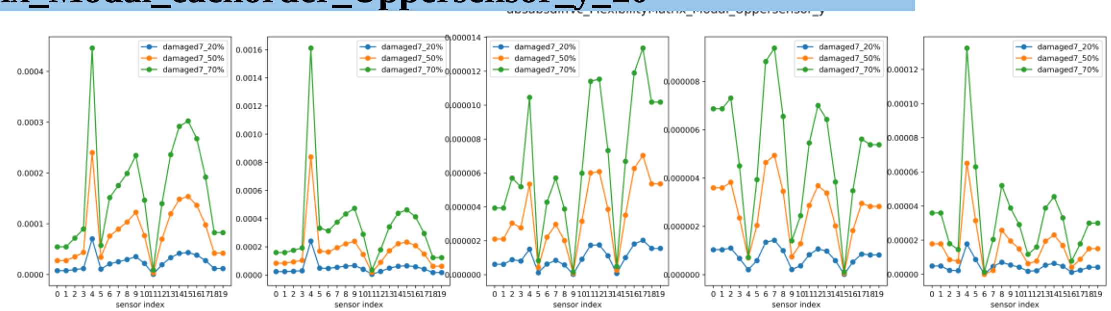
$\mathbf{v}_{\text{undamaged}}$
$\mathbf{v}_{\text{damaged}}$

$$\begin{bmatrix} a_{11} + \cdots + a_{1n} \\ \vdots \\ a_{n1} + \cdots + a_{nn} \end{bmatrix}_{n\times 1} \begin{bmatrix} a_{11} + \cdots + a_{1n} \\ \vdots \\ a_{n1} + \cdots + a_{nn} \end{bmatrix}_{n\times 1} \dots \begin{bmatrix} a_{11} + \cdots + a_{1n} \\ \vdots \\ a_{n1} + \cdots + a_{nn} \end{bmatrix}_{n\times 1}$$

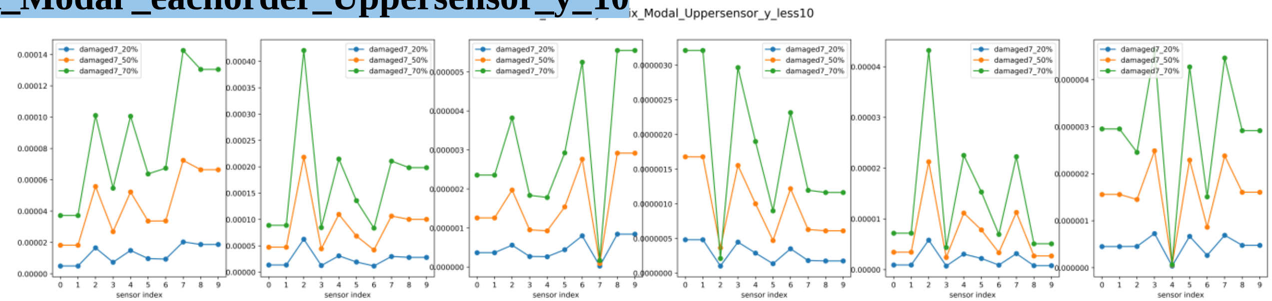
$\mathbf{v}_{\text{curvature\_undamaged}}$
$\mathbf{v}_{\text{curvature\_damaged}}$

$$\left.\begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}_{n\times 1} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}_{n\times 1} \dots \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix}_{n\times 1} \right\} \text{DamageIndex:Diff\_}\mathbf{vc}$$

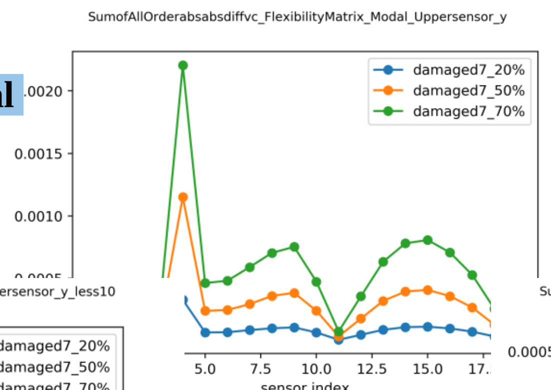
Absabsdiff_FlexibilityMatrix_Modal_eachorder_Uppersensor_y_20



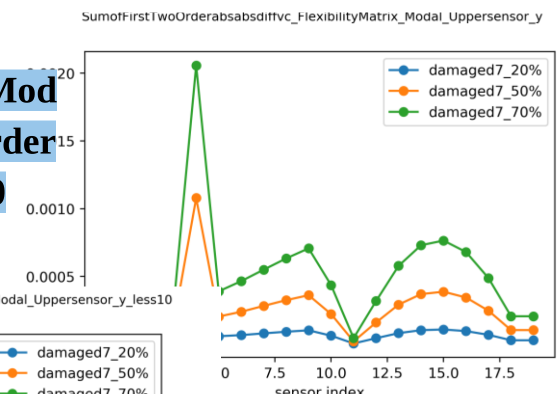
Absabsdiff_FlexibilityMatrix_Modal _eachorder_Uppersensor_y_10

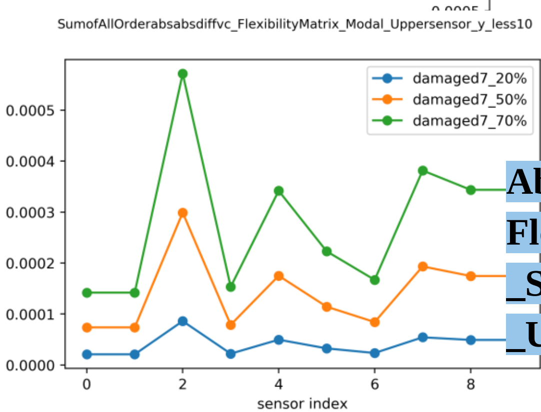


Absabsdiff_ FlexibilityMatrix_Modal _SumAllOrder _Uppersensor_y_20

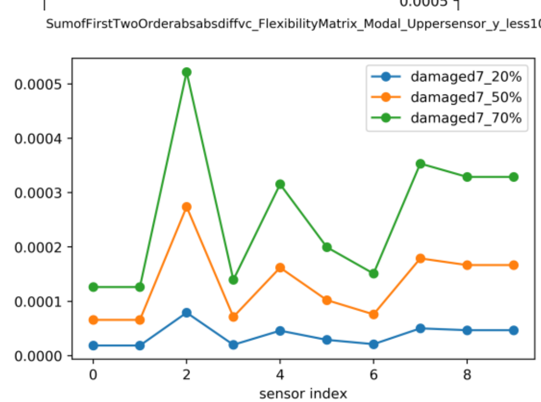


Absabsdiff_ FlexibilityMatrix_Mod al _SumOfFirst2Order _Uppersensor_y_20



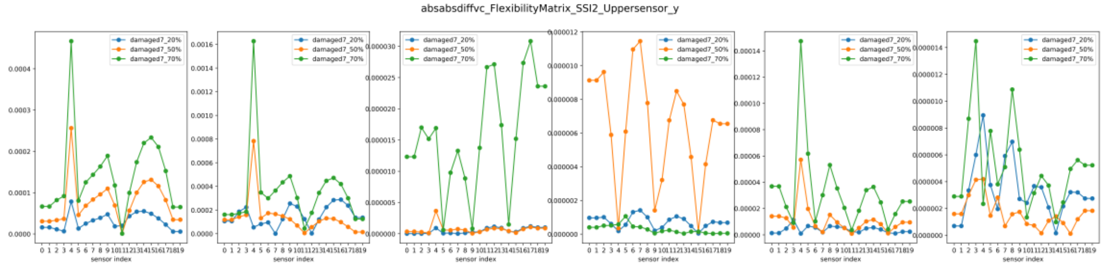


Absabsdiff_ FlexibilityMatrix_Modal _SumAllOrder _Uppersensor_y_10

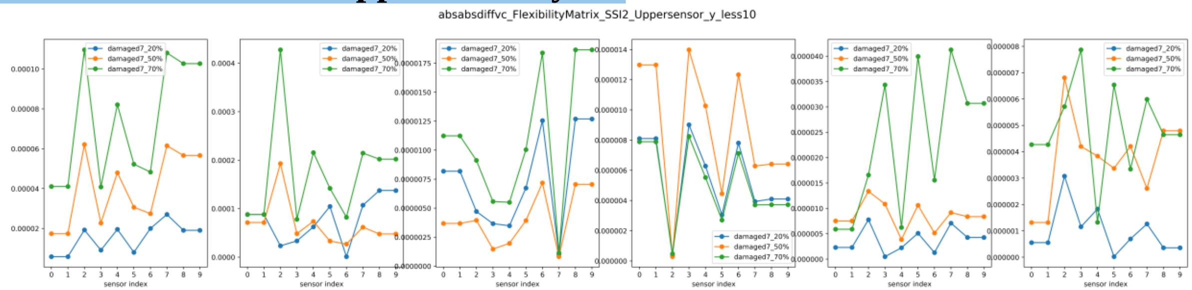


Absabsdiff_ FlexibilityMatrix_Modal _SumOfFirst2Order _Uppersensor_y_10

Absabsdiff_FlexibilityMatrix_SSI2_eachorder_Uppersensor_y_20



Absabsdiff_FlexibilityMatrix_SSI2 _eachorder_Uppersensor_y_10



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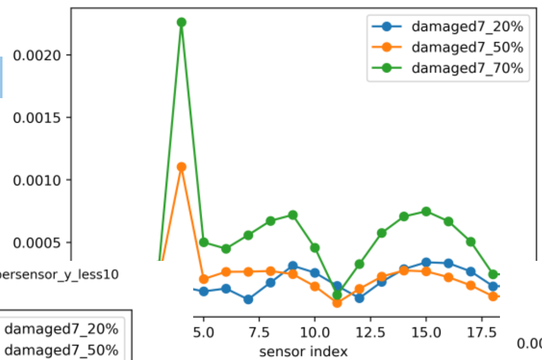

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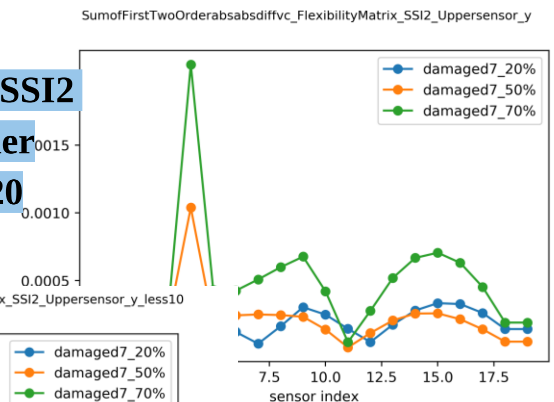
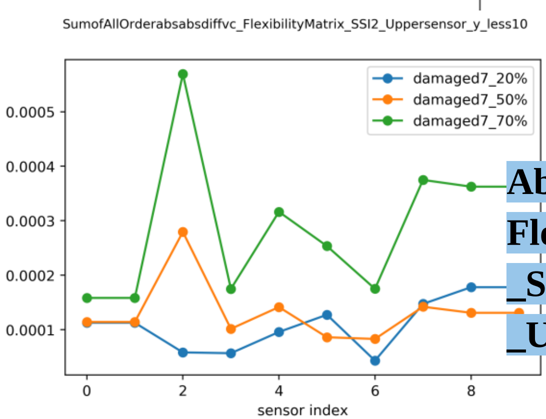
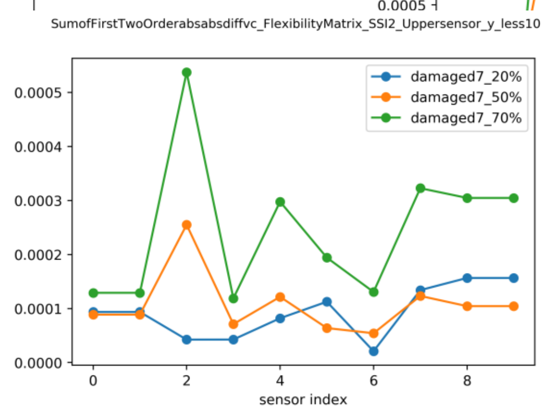




Absabsdiff_ FlexibilityMatrix_SSI2 _SumAllOrder _Uppersensor_y_10



Absabsdiff_ FlexibilityMatrix_SSI2 _SumOfFirst2Order _Uppersensor_y_10

Part 4 Summary

Mode Shape Curvature:

+ **easy to operate, informative result**
+ **only needs consistent normalized mode shape**
- **higher order not very informative**

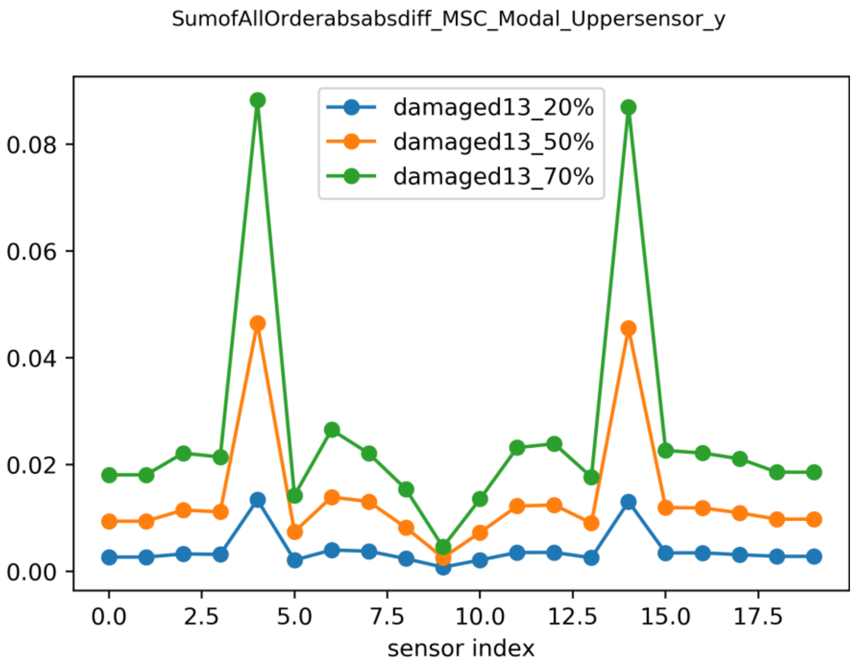
Modal Strain Energy:

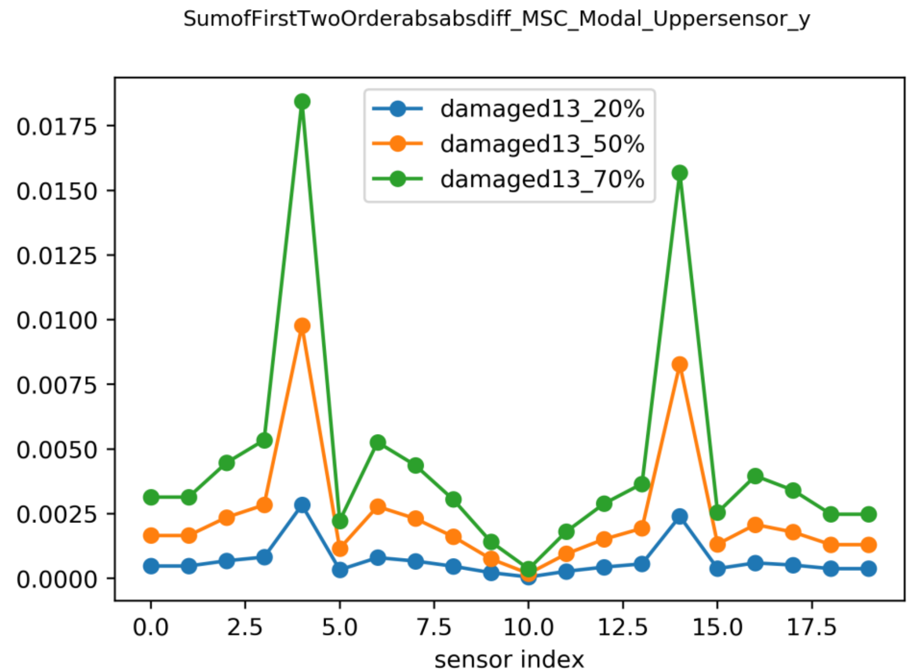
+ **only needs consistent normalized mode shape**
- **higher order not very informative**

Flexibility Matrix:

+ **Higher order mode should be informative**
- **need mass normalized mode shape which is not easy to obtain**

- **One step further**
- Simultaneously apply damaged 7 and damaged 12 (we call it damage 13)
- Mode Shape Curvature
- Modal Strain energy
- Flexibility matrix


SumofAllOrderabsabsdiff_MSC_Modal_Uppersensor_y
damaged13_20%
damaged13_50%
damaged13_70%
0.00
0.02
0.04
0.06
0.08
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
sensor index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damaged13_50%
damaged13_70%
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0.0025
0.0050
0.0075
0.0100
0.0125
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0.0175
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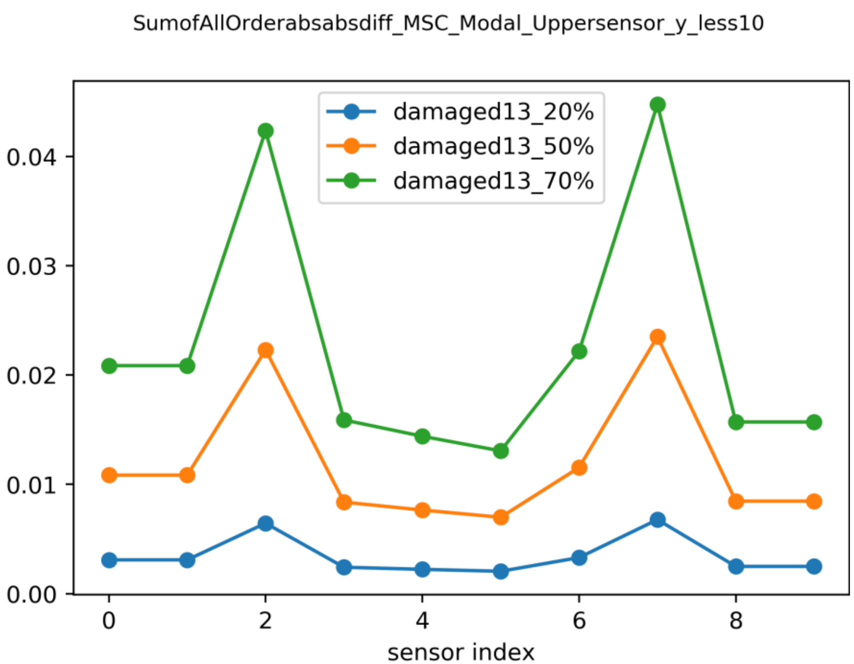
10.0
12.5
15.0
17.5
sensor index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0.04
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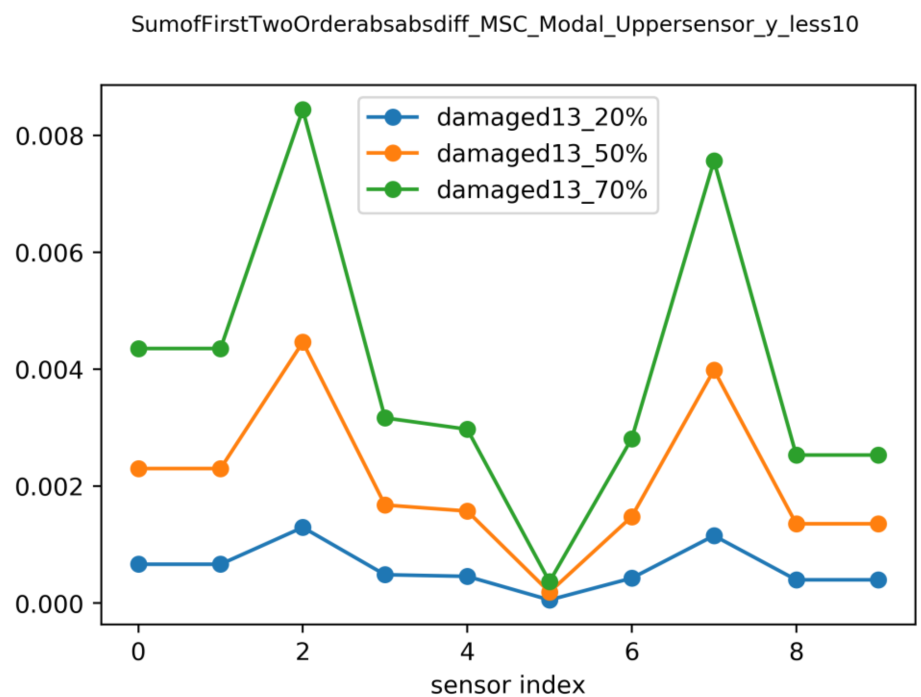
2
4
6
8
sensor index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0.008
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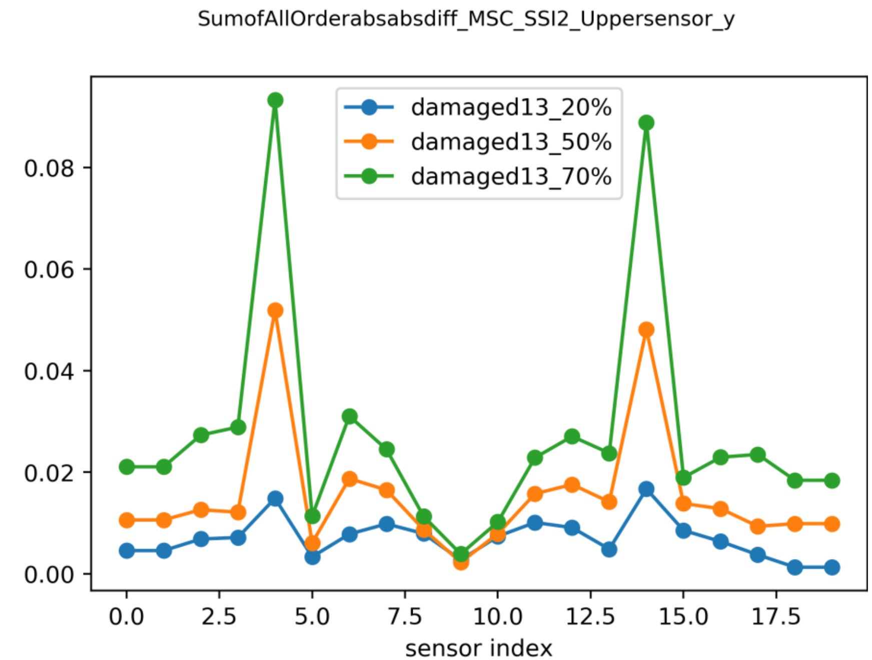
4
6
8
sensor index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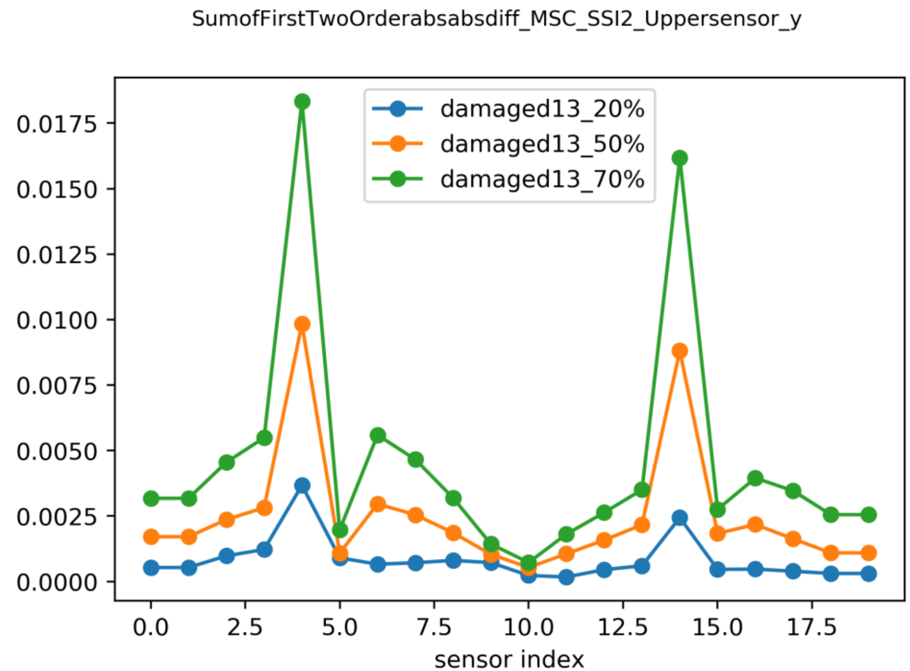
sensor index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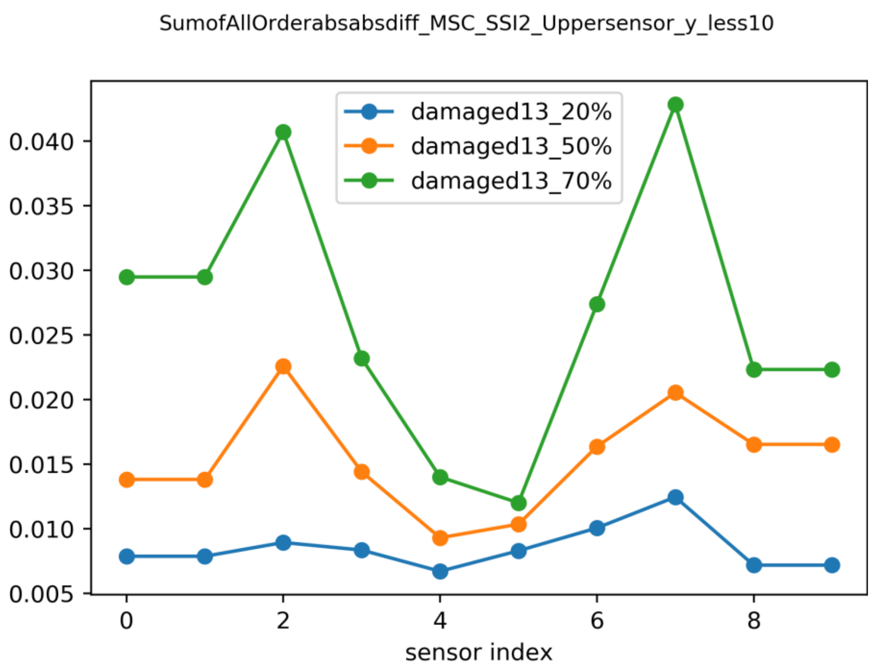
sensor index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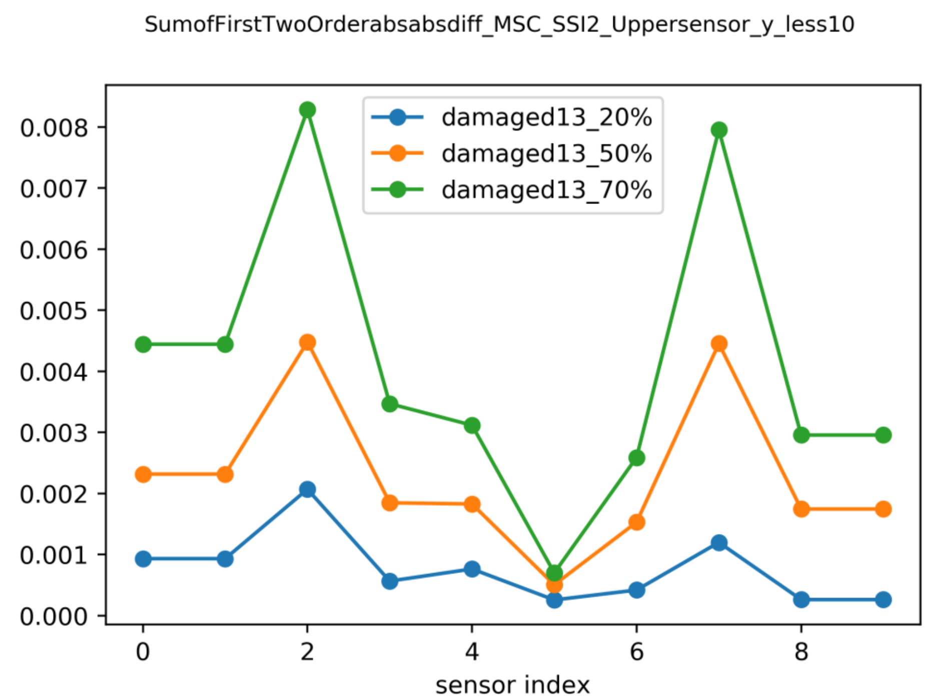
sensor index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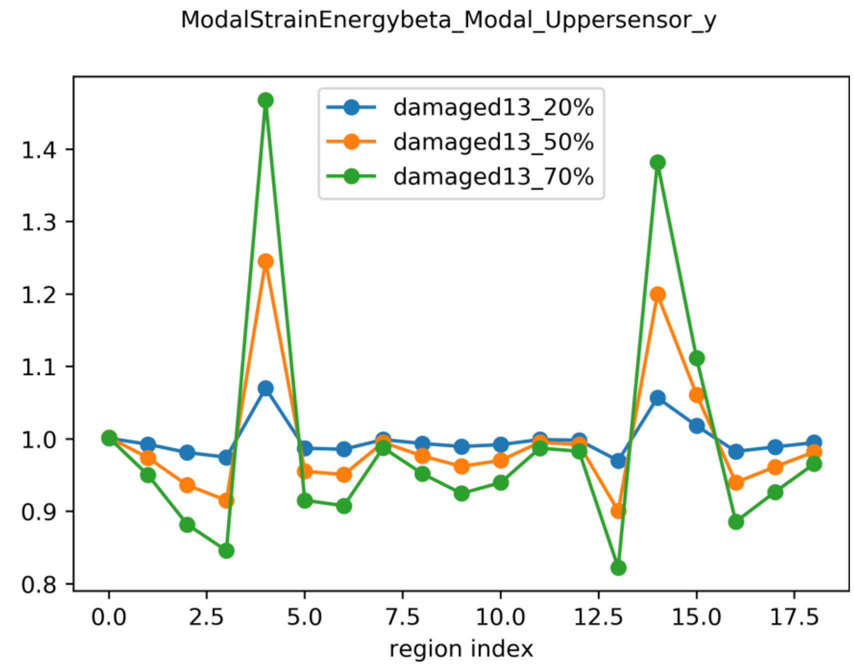
sensor index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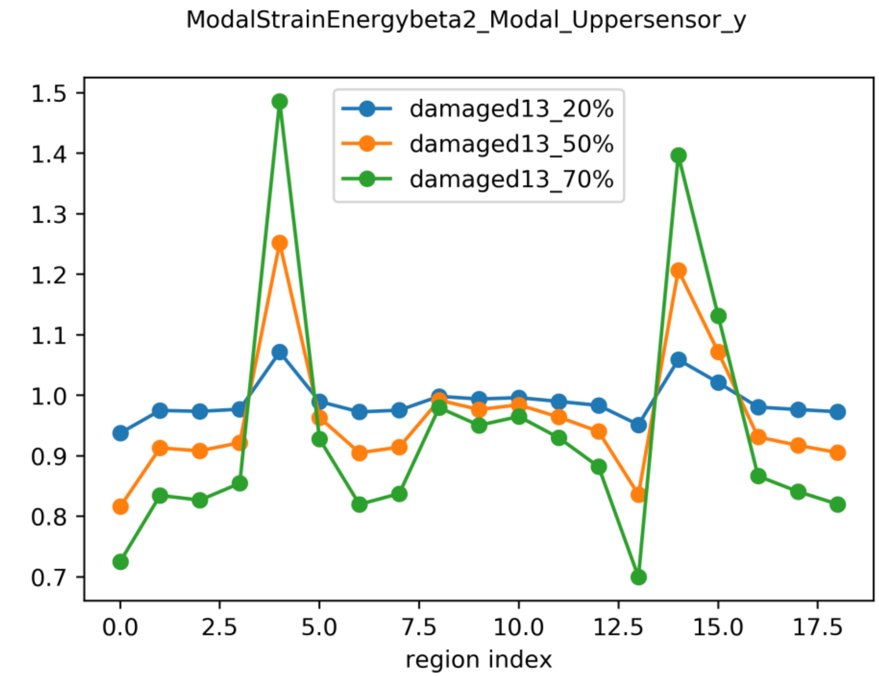
region index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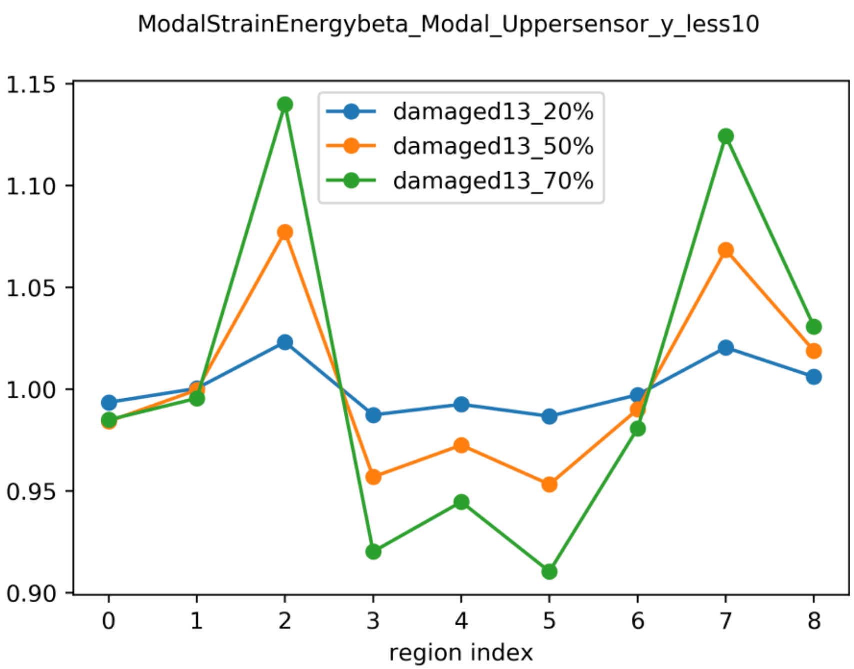
region index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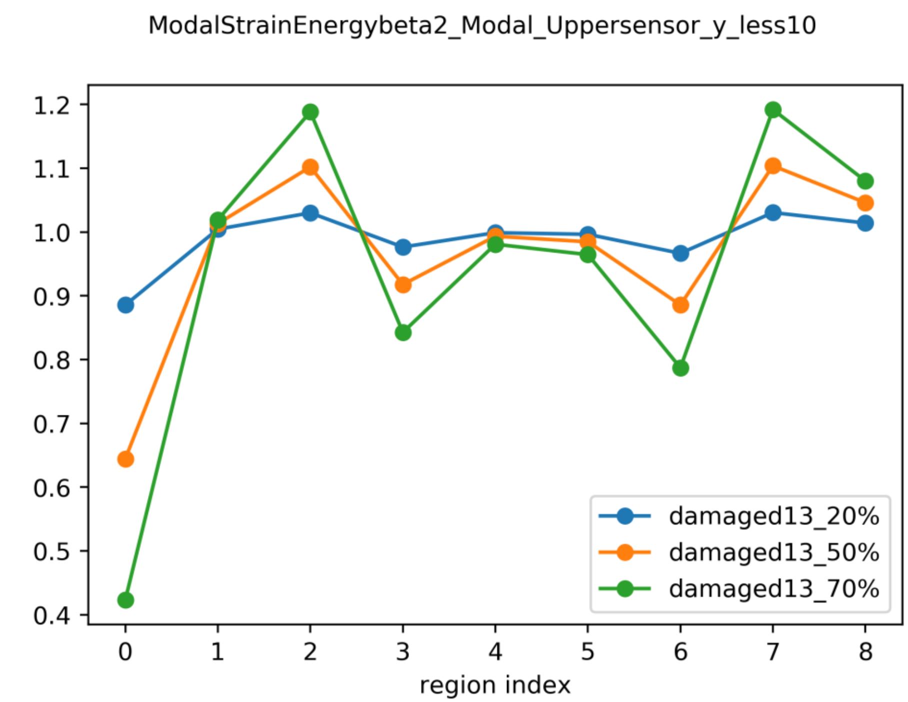
region index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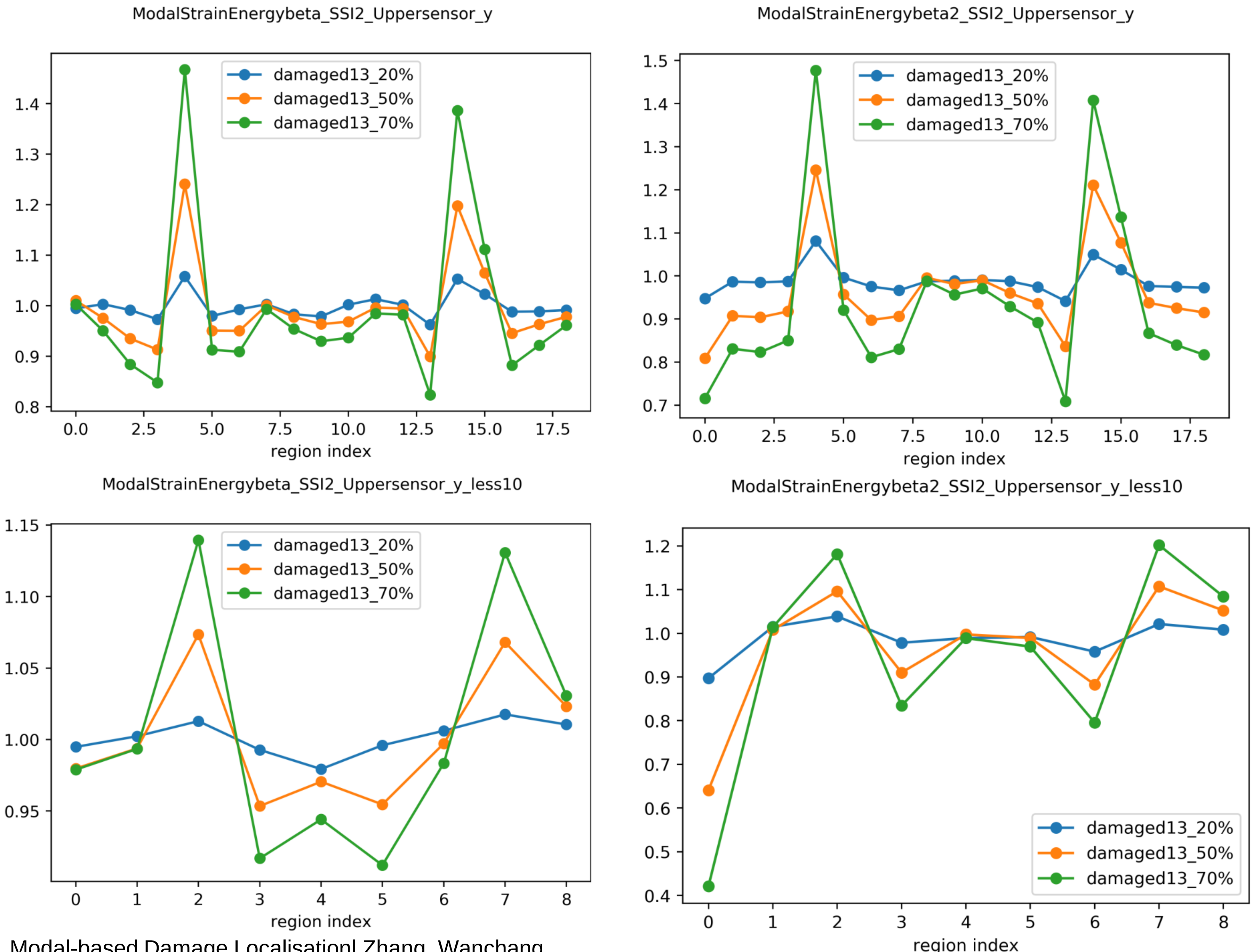
damaged13_20%
damaged13_50%
damaged13_70%
region index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damaged13_50%
damaged13_70%
region index
ModalStrainEnergybeta2_SSI2_Uppersensor_y
damaged13_20%
damaged13_50%
damaged13_70%
region index
ModalStrainEnergybeta_SSI2_Uppersensor_y_less10
damaged13_20%
damaged13_50%
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region index
ModalStrainEnergybeta2_SSI2_Uppersensor_y_less10
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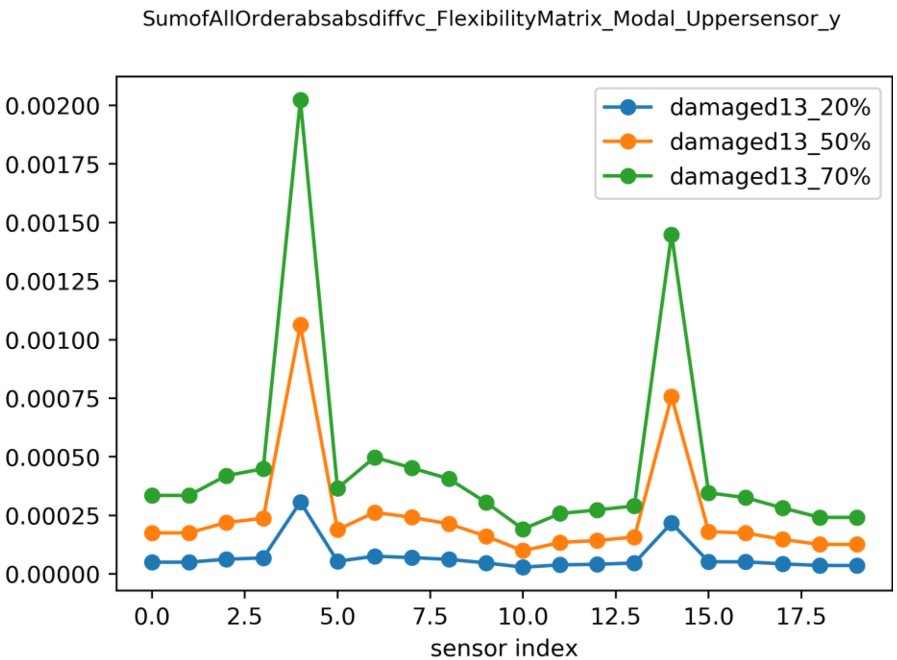
damaged13_50%
damaged13_70%
region index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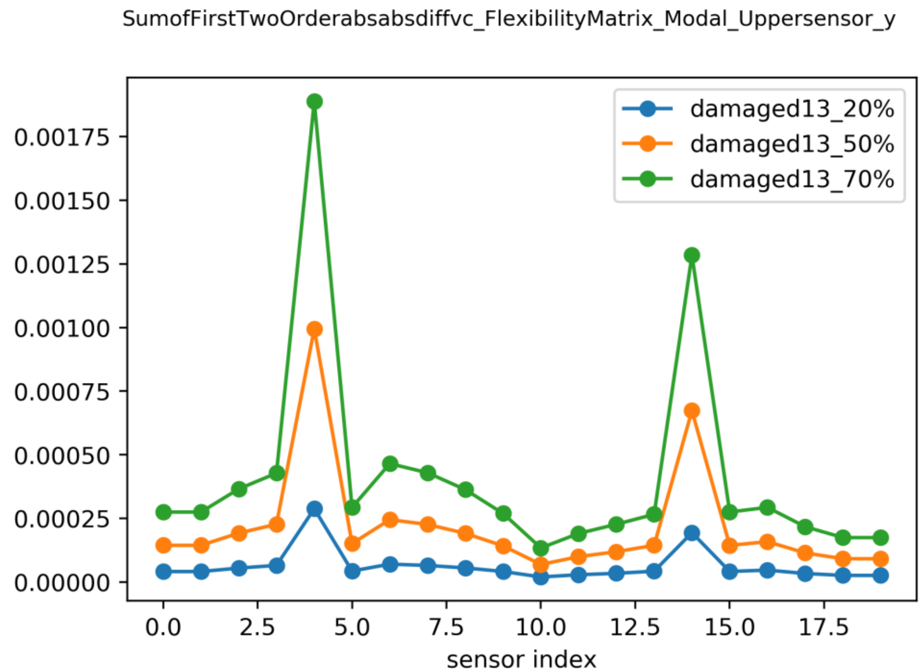
damaged13_50%
damaged13_70%
sensor index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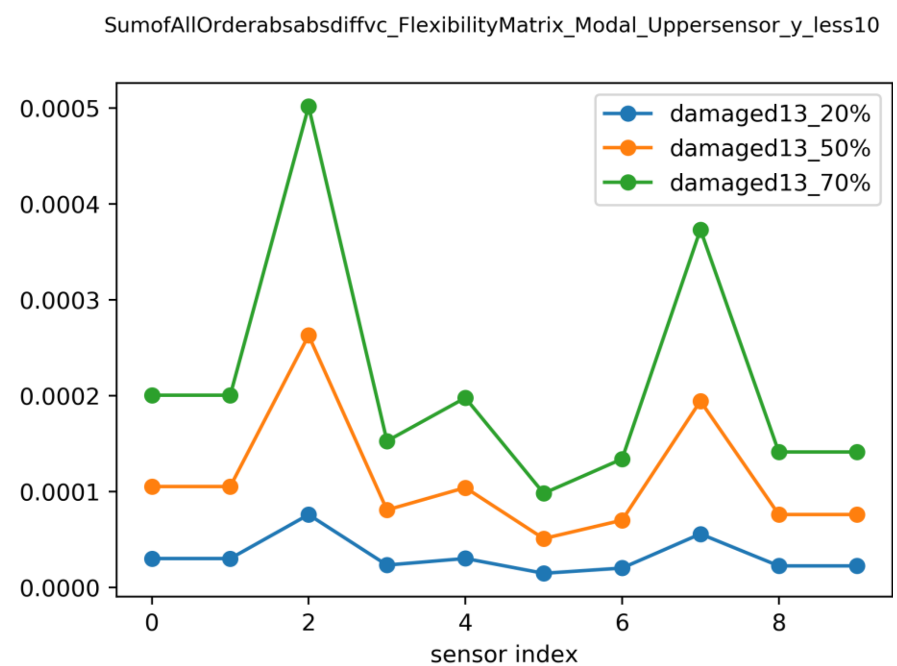
sensor index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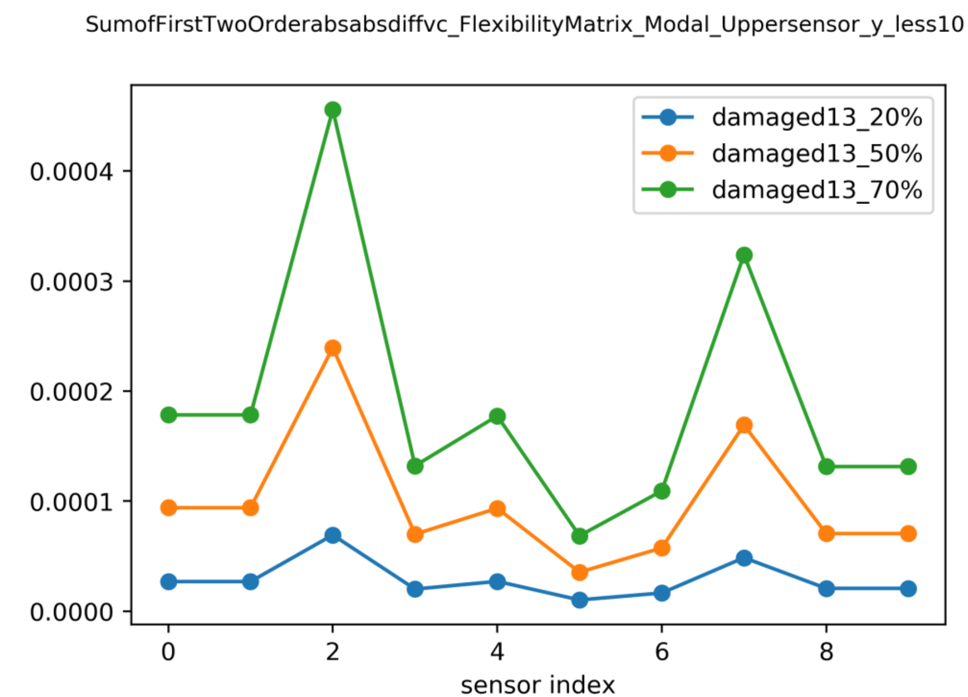
damaged13_50%
damaged13_70%
sensor index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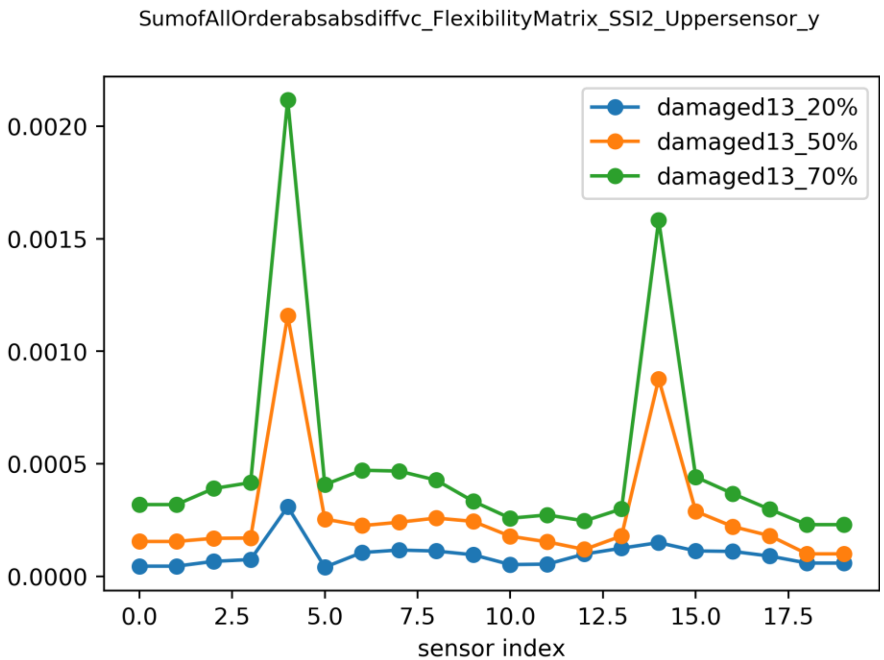
sensor index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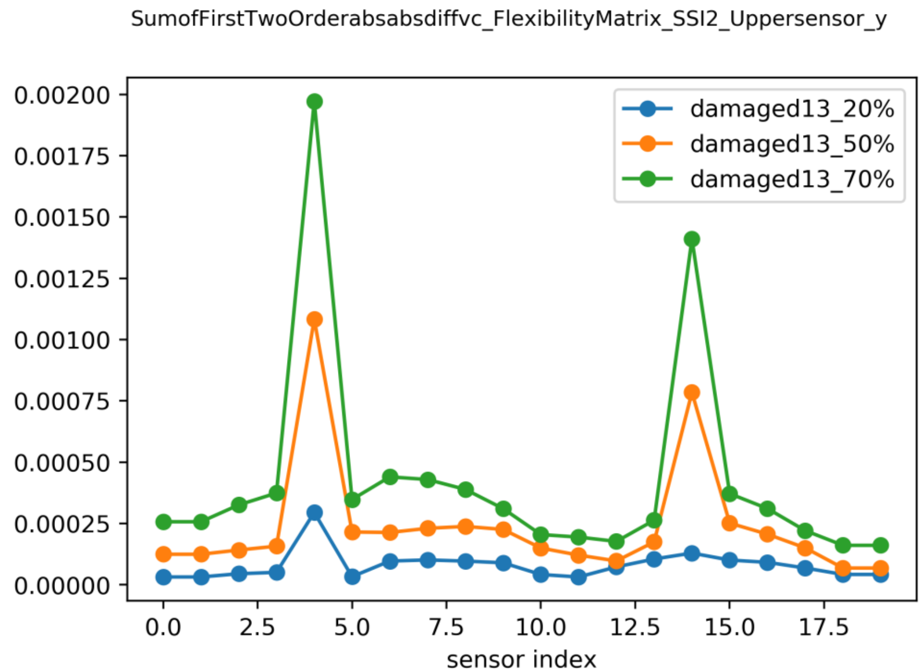
damaged13_50%
damaged13_70%
sensor index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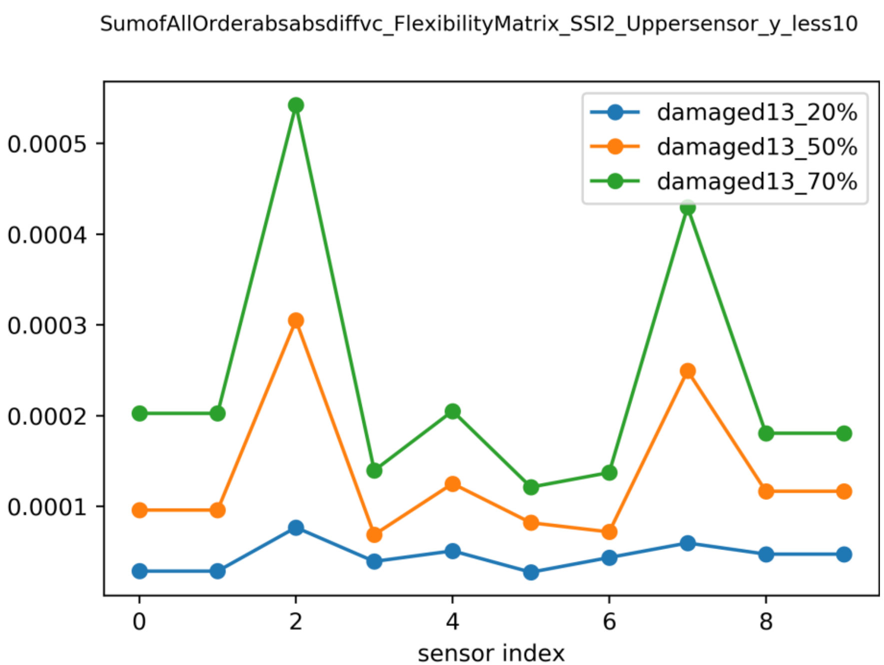
damaged13_70%
sensor index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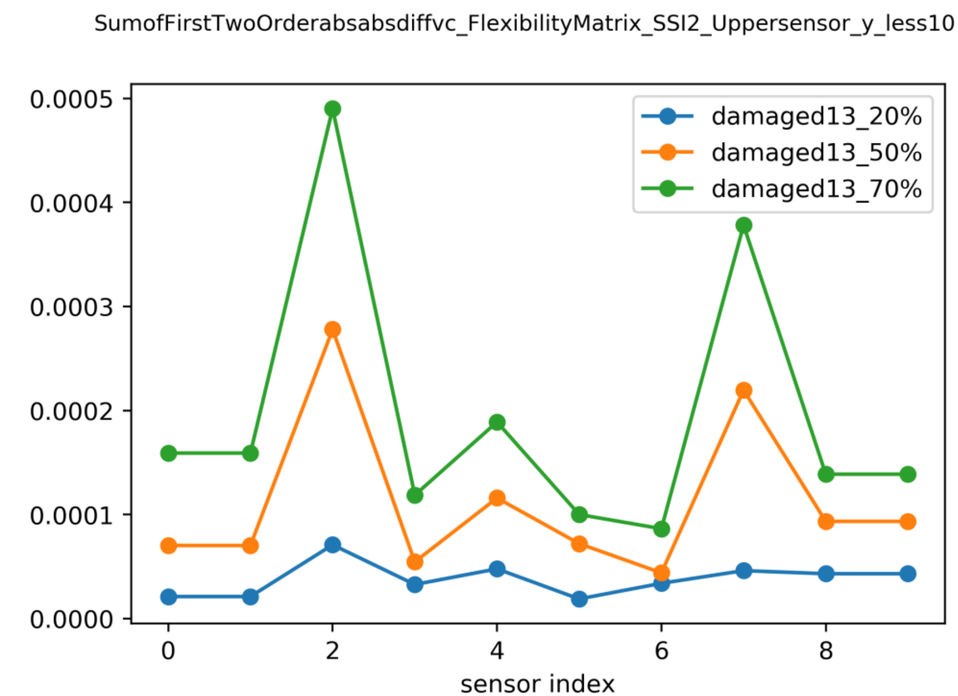
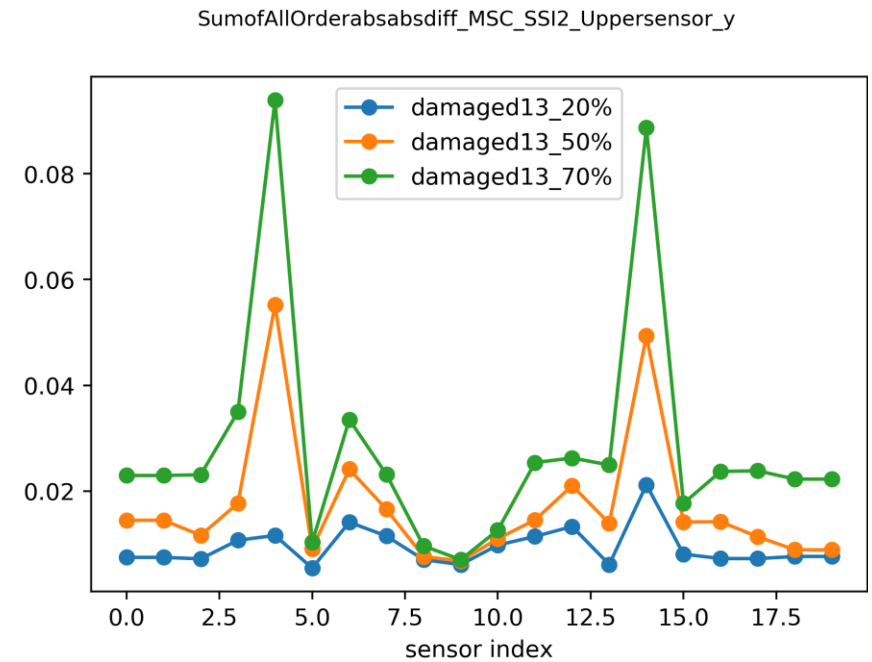
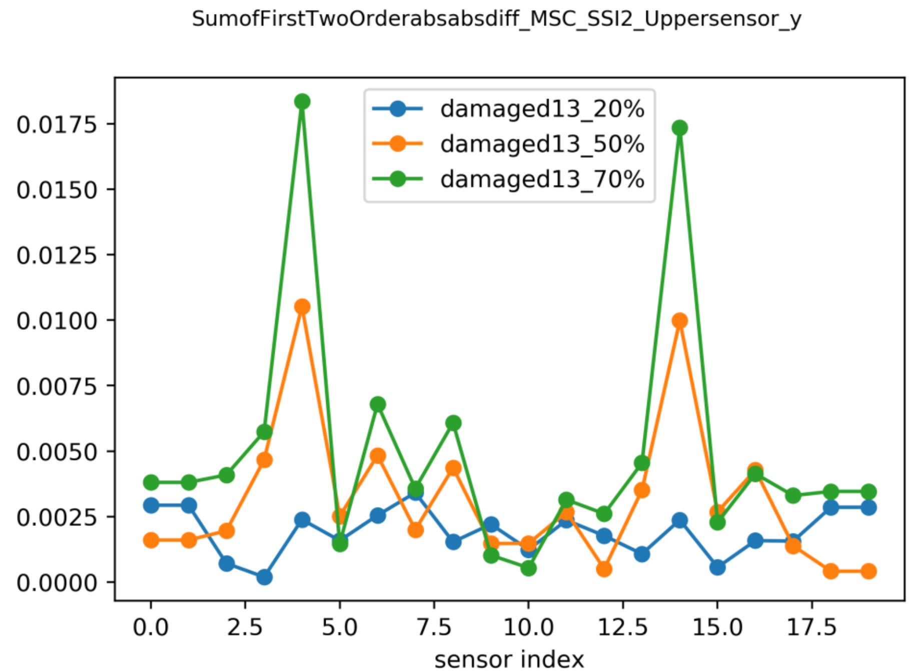
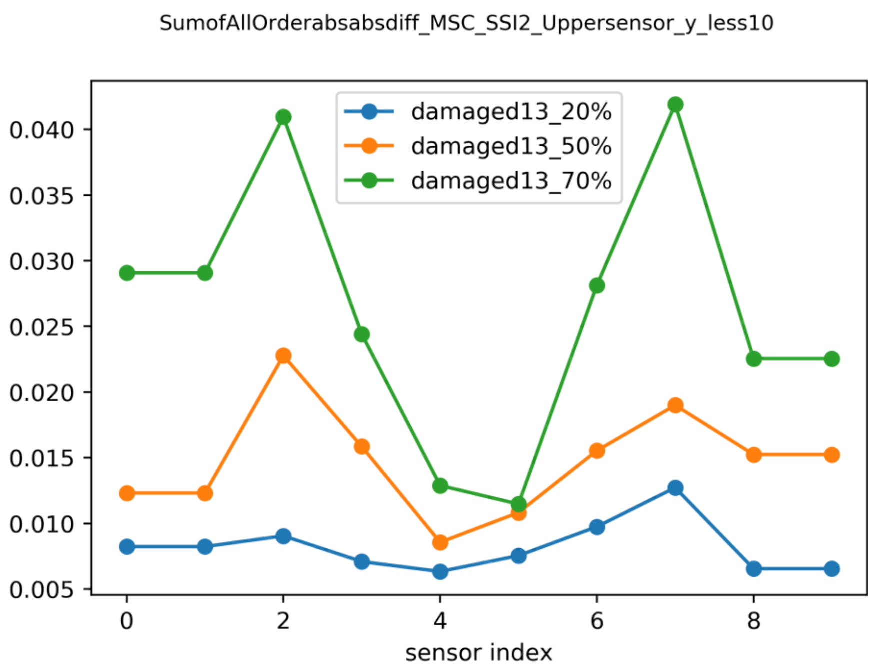
damaged13_20%
damaged13_50%
damaged13_70%
sensor index

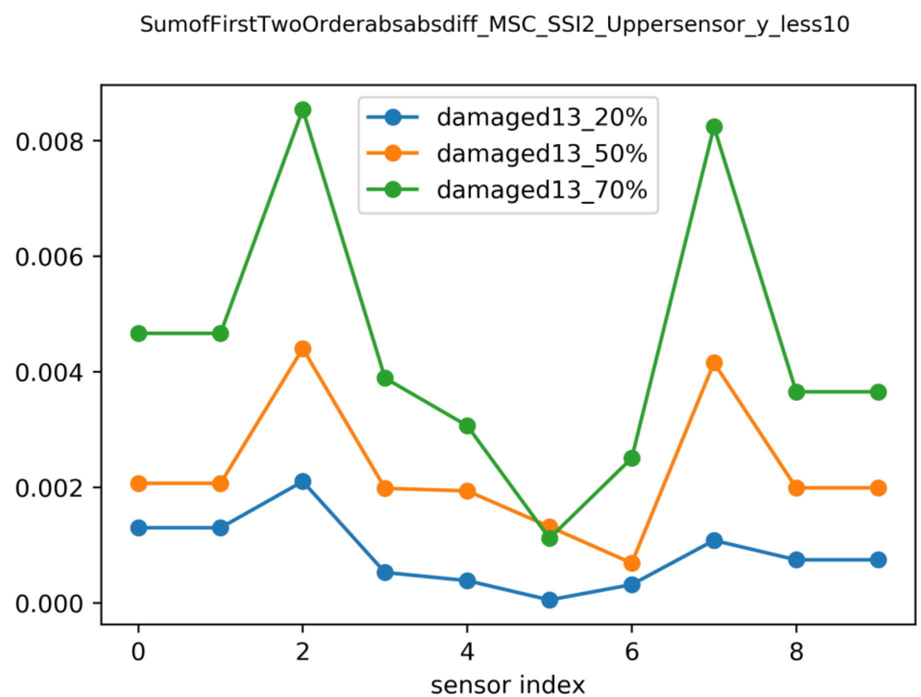

SumofFirstTwoOrderabsabsdiffvc_FlexibilityMatrix_SSI2_Uppersensor_y_less10
damaged13_20%
damaged13_50%
damaged13_70%
sensor index

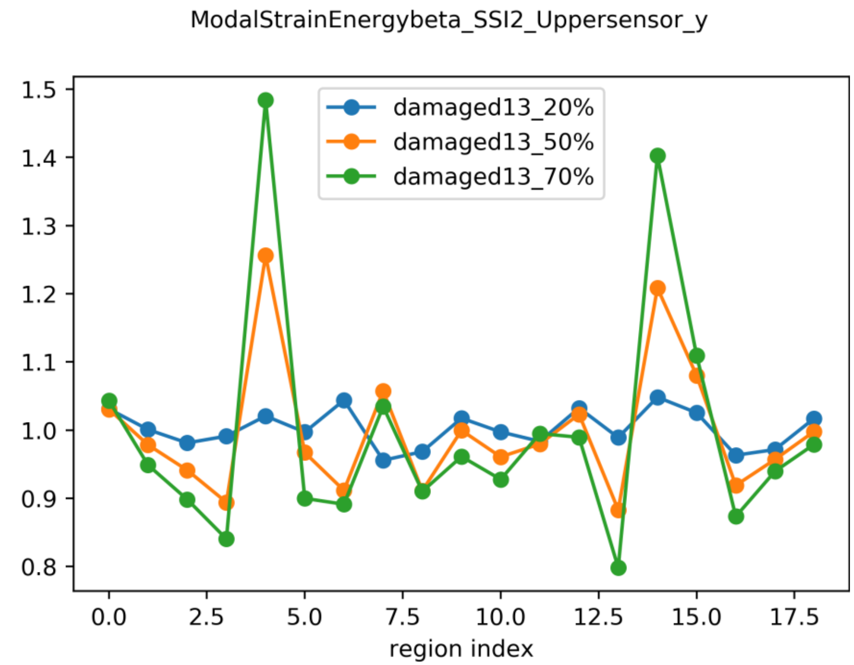
- **Two steps further**
- Simultaneously apply damaged 7 and damaged 12 (we call it damage 13)
- Add the 5% noise when obtaining the SSI data
- Mode Shape Curvature
- Modal Strain energy
- Flexibility matrix

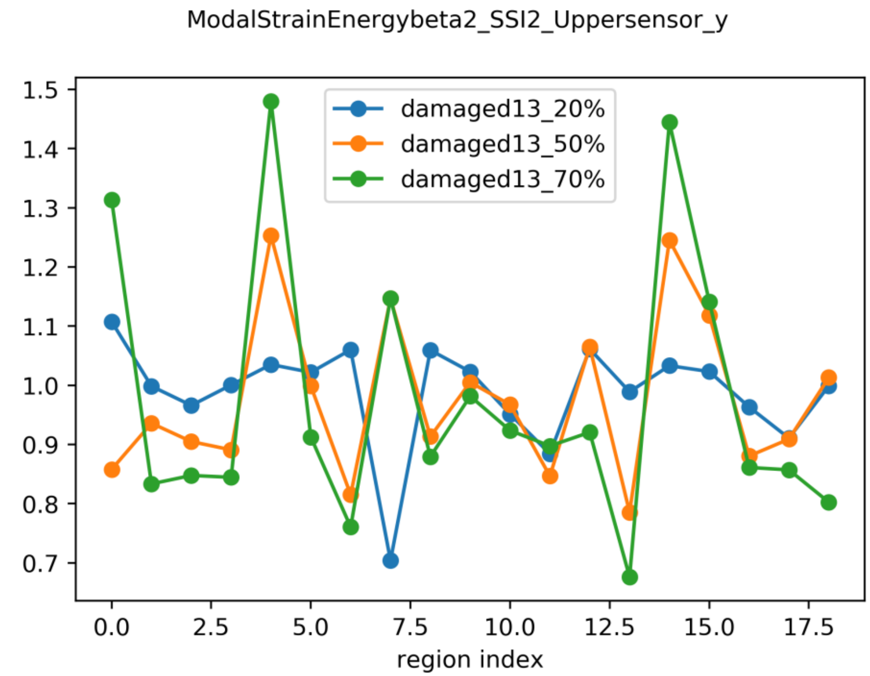

SumofAllOrderabsabsdiff_MSC_SSI2_Uppersensor_y
damaged13_20%
damaged13_50%
damaged13_70%
sensor index

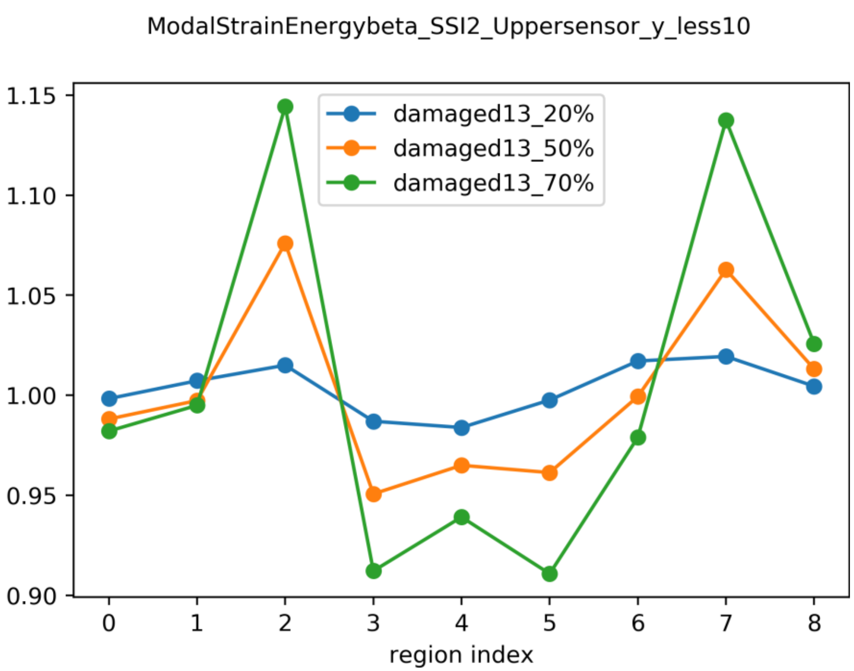

SumofFirstTwoOrderabsabsdiff_MSC_SSI2_Uppersensor_y
damaged13_20%
damaged13_50%
damaged13_70%
sensor index

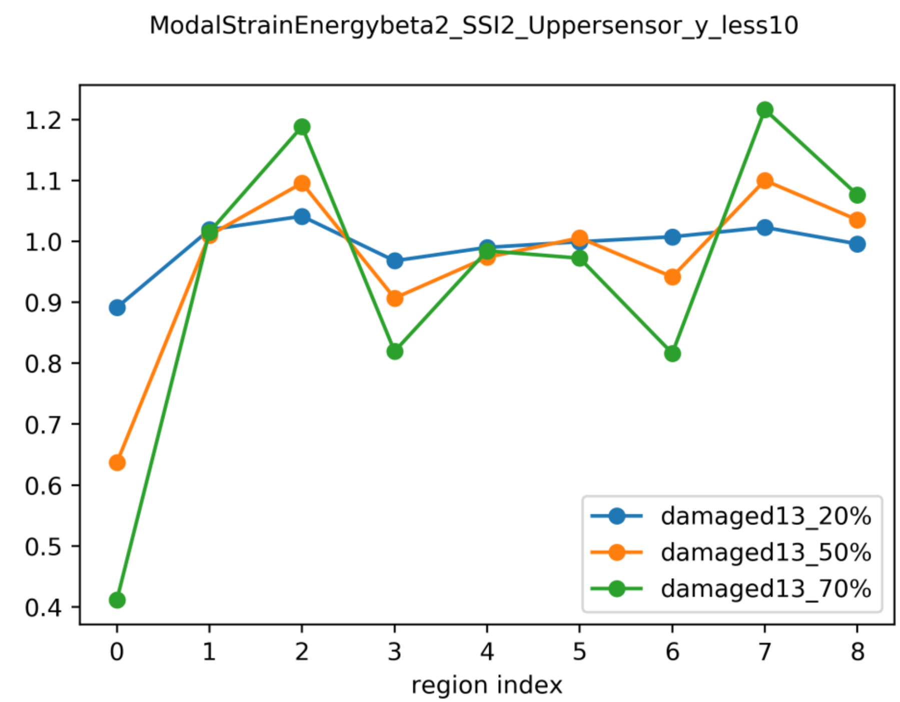

SumofAllOrderabsabsdiff_MSC_SSI2_Uppersensor_y_less10
damaged13_20%
damaged13_50%
damaged13_70%
sensor index

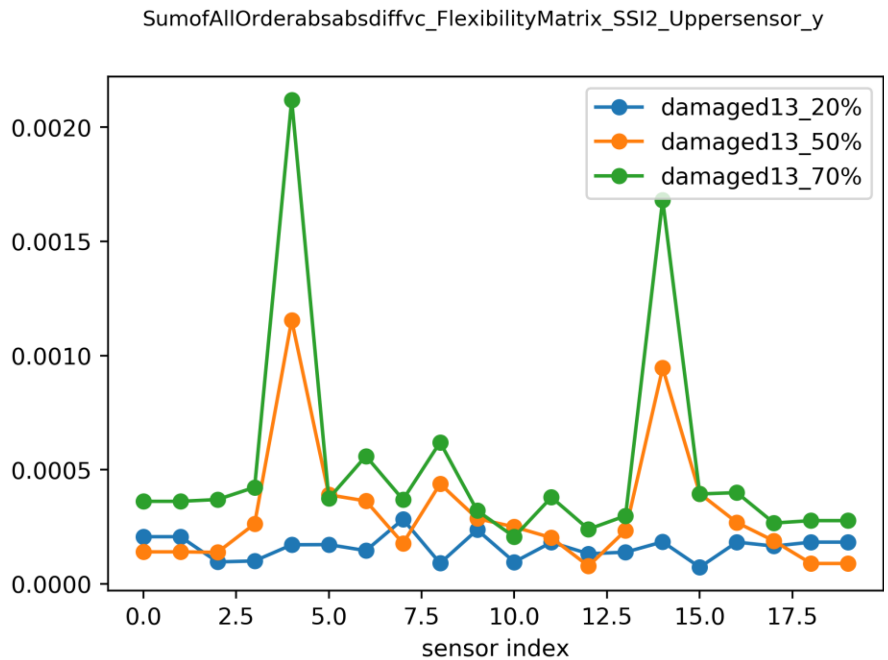

SumofFirstTwoOrderabsabsdiff_MSC_SSI2_Uppersensor_y_less10
damaged13_20%
damaged13_50%
damaged13_70%
sensor index

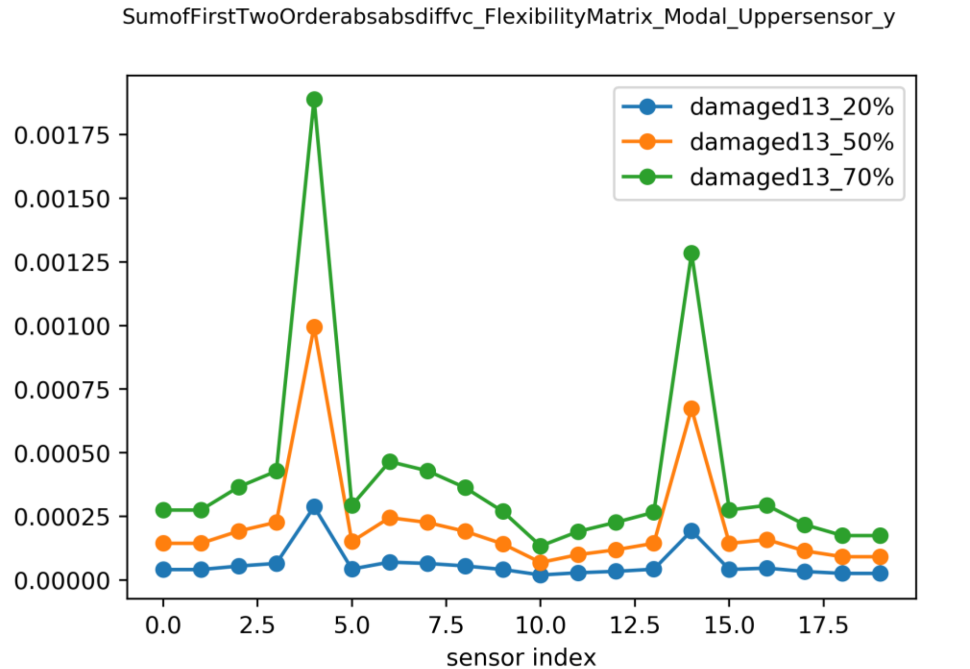

ModalStrainEnergybeta_SSI2_Uppersensor_y
damaged13_20%
damaged13_50%
damaged13_70%
region index

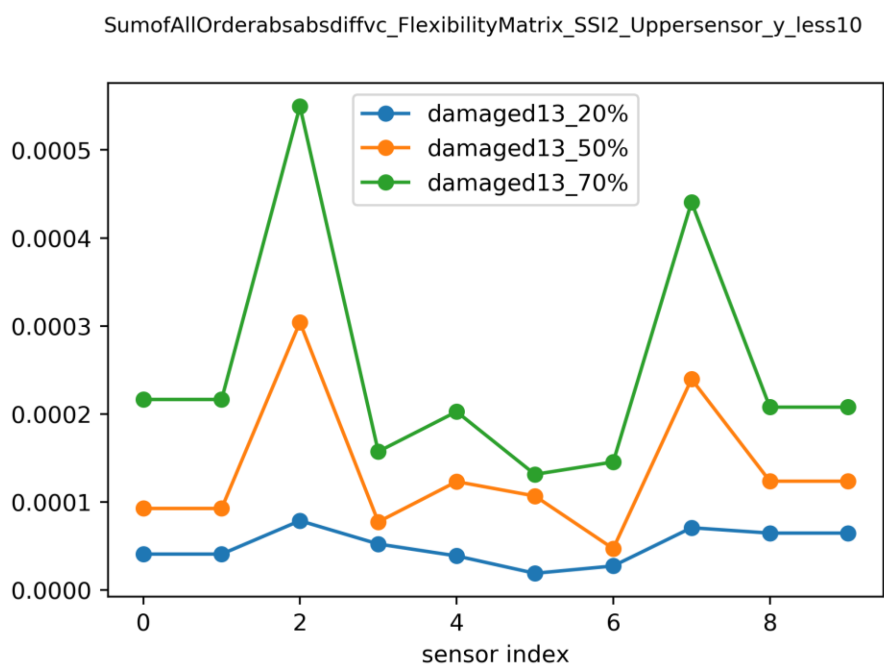

ModalStrainEnergybeta2_SSI2_Uppersensor_y
damaged13_20%
damaged13_50%
damaged13_70%
region index

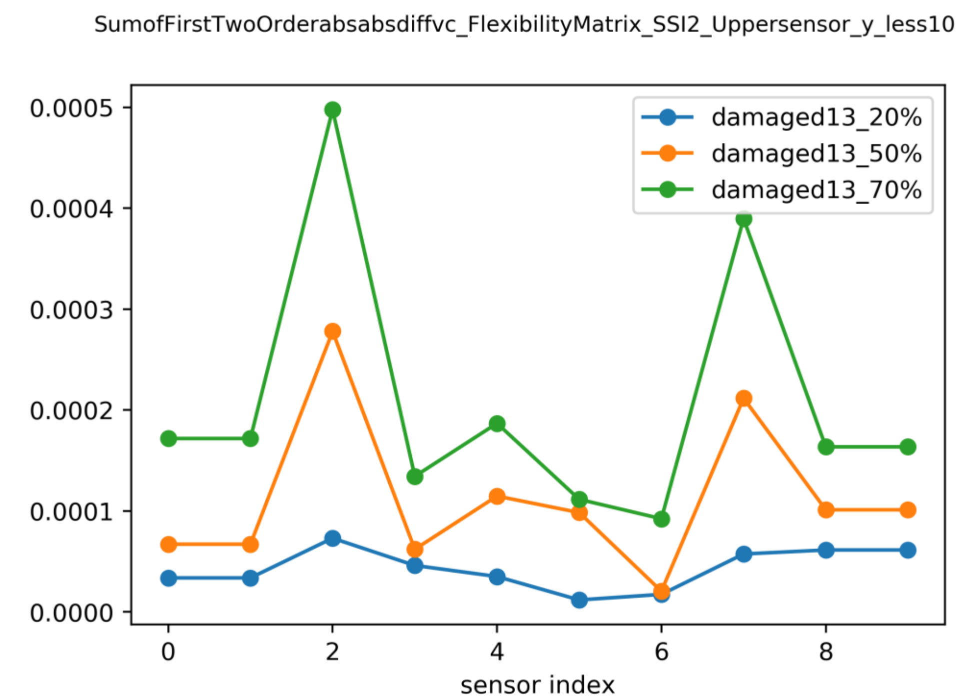

ModalStrainEnergybeta_SSI2_Uppersensor_y_less10
damaged13_20%
damaged13_50%
damaged13_70%
region index


ModalStrainEnergybeta2_SSI2_Uppersensor_y_less10
damaged13_20%
damaged13_50%
damaged13_70%
region index


SumofAllOrderabsabsdiffvc_FlexibilityMatrix_SSI2_Uppersensor_y
damaged13_20%
damaged13_50%
damaged13_70%
0.0000
0.0005
0.0010
0.0015
0.0020
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
sensor index


SumofFirstTwoOrderabsabsdiffvc_FlexibilityMatrix_Modal_Uppersensor_y
damaged13_20%
damaged13_50%
damaged13_70%
0.00000
0.00025
0.00050
0.00075
0.00100
0.00125
0.00150
0.00175
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
sensor index


SumofAllOrderabsabsdiffvc_FlexibilityMatrix_SSI2_Uppersensor_y_less10
damaged13_20%
damaged13_50%
damaged13_70%
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
0
2
4
6
8
sensor index


SumofFirstTwoOrderabsabsdiffvc_FlexibilityMatrix_SSI2_Uppersensor_y_less10
damaged13_20%
damaged13_50%
damaged13_70%
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
0
2
4
6
8
sensor index